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HIGH FREQUENCY PROPERTIES OF PLASMA

K. D. Sinel'nikov, Editor-in-Chief

Academy of Sciences, Ukrainian SSR,
Izdatel'stvo "Naukova Dumka,"
Kiev, 1965

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TABLE OF CONTENTS

HIGH FREQUENCY PROPERTIES OF PLASMA

H I G H   F R E Q U E N C Y   P R O P E R T I E S   O F   P L A S M A

PLASMA PHYSICS AND THE PROBLEMS OF CONTROLLED THERMONUCLEAR REACTION

ABSTRACT

These articles present the results derived from theoretical and experimental investigations of high frequency plasma properties: the methods of high frequency plasma heating, propagation of electromagnetic waves in a magnetoactive plasma, thermal radiation of a plasma, and development of instabilities when employing high frequency methods of plasma heating. A description is provided of experimental equipment developed for high frequency heating and containment of a plasma.

This collection is designed for scientific researchers and engineers dealing with the problems of a plasma and its technical application, as well as for students and graduate students in the physics departments of universities and physical-technical institutes.

SECTION I

HIGH FREQUENCY PLASMA HEATING

INVESTIGATION OF THE ENERGY OF CHARGED PARTICLES EMANATING FROM A MAGNETIC TRAP DURING HIGH FREQUENCY HEATING

N. I. Nazarov, A. I. Yermakov, V. T. Tolok

The methods of resonance excitation of the eigen fluctuations of a plasma cylinder by outer electromagnetic fields have been extensively employed in plasma heating with high frequency fields. Spatially periodic electromagnetic fields may be employed to excite the eigen fluctuations in a plasma located in a constant magnetic field to frequencies which are close /5*

* Note: Numbers in the margin indicate pagination in the original foreign text.

to the gyrofrequency of ions $\omega \approx \omega_{Hi}$ (an ion cyclotron wave) or to frequencies $\omega_{Hi} < \omega \ll \omega_{He}$ (rapid magnetosound wave), where ω_{He} is the electron gyrofrequency and ω_{Hi} is the ion gyrofrequency. In the first case, the energy of electromagnetic fluctuations is transmitted directly to the ions, and in the second case the energy is transmitted to the plasma electrons.

The articles (Ref. 1, 2) have investigated the conditions for the resonance excitation of these fluctuations, their propagation, and damping. It has been found that even at high electron temperatures the damping is significant and is caused by a collisionless mechanism. It was also found that, /6
when this excitation method is employed, high frequency power is transmitted to the plasma very effectively.

This study presents the results derived from measuring the energy of ions and electrons passing along the magnetic field, when ion cyclotron and rapid magnetohydrodynamic waves are employed for heating the plasma. The experiment was performed on a "Sneg" apparatus, which has been described in great detail previously (Ref. 1). The eigen fluctuations were excited in the plasma by spatially periodic electromagnetic fields at a frequency of 10 Mc with the appropriate selection of the magnetic field strength H_0. In contrast to preceding experiments, the power of the high frequency (hf) generator was increased to 300 kw.

In order to increase the power introduced into the plasma, the pulse of the hf generator was programmed so that the strong loading on the circuit at the moment of its resonance loading by the plasma was compensated by a corresponding voltage increase in the pulse from the hf generator (Figure 1). Thus, the necessity of a special electrical strengthening of the hf circuit was avoided, even when a power greater than 100 kw was introduced into the plasma.

The energy of the charged particles was measured by a transit time electrostatic analyzer (Ref. 3) and a multigrid probe (Ref. 4). The first method made it possible to study the energy spectrum and the mass composition of plasma ions; the second method made it possible to measure the energy of ions and electrons. The plasma electron temperature was determined by a spectral method.

The input slit of the analyzer, which was located 25 cm behind a magnetic mirror, cut out a narrow plasma flux, from which an ion bundle was separated after passing a separation device. The energy of the bundle was analyzed by the electric field of the flat condenser. Ion fluxes were recorded by an ion-electron converter, which changed the ion bundle into a bundle of electrons accelerated up to 20 kev, after deflection in the analyzing condenser. These electrons were detected by a plastic scintillator with a photoelectric multiplier.

In order to study the mass composition of the plasma ions, the flight

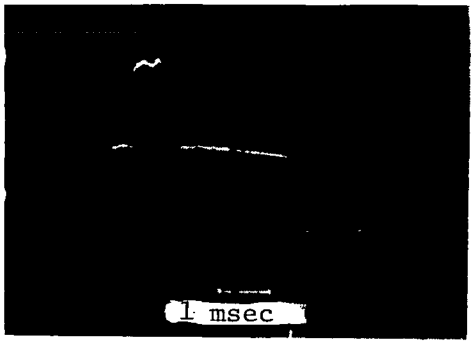

Figure 1

time was measured by ions having a drift section 56 cm long. When the analyzer modulator was supplied with voltage having a rectangular form, it was possible to obtain short pulses of the ion current (τ = 1; 0.5; 0.2 microseconds). Due to a difference in the velocities of different ions, the ions were separated by mass in the drift space. The flight time of ions in the drift sec- /7
tion was used to determine the ion velocities and masses, respectively. By measuring the amplitude of the current signals at different periods of time and by changing the voltage on the analyzing condenser, it was possible to record the energy spectra of ions having different masses and to observe the change in ion energy during heating, by means of this analyzer.

In order to exclude the scatter of ion current pulses, the result was averaged over ten measurements (the ion energy was determined with the analyzer with an accuracy of 8%).

Figure 2 presents the oscillograms of a typical signal from the photomultiplier, whose magnitude was proportional to the current of ions having an energy of 1500 ev — a; b — represents the suppression signal of microwaves having a wavelength of 8 mm.

Figure 3 shows the energy spectra of plasma protons (the coordinates: the distribution functions ψ -- ion energy E_i) when the plasma is heated by means of ion cyclotron waves for two voltages on the hf circuit (curve 1 -- U_c = 28 kv; curve 2 -- U_c = 32 kv). The plasma density to be measured during heating was no less than $2 \cdot 10^{13}$ cm^{-3}. As may be seen from the figure, the energy at the spectrum maximum amounts to 2 kev. With an increase in the voltage U_c on the exciting coil of the hf circuit, the mean ion energy in the spectrum increases proportionally to the square of the voltage U_c^2. However, during the heating pulse it remains almost constant, which points to /8
large losses which are apparently caused by overcharging.

For purposes of comparison, the proton energy spectrum is presented which was recorded when the plasma was heated by a rapid magnetosound wave with a

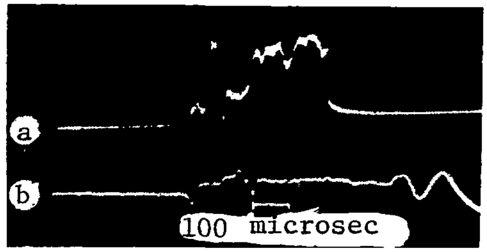

Figure 2

voltage of 32 kv on the hf circuit (Figure 4). The ion energy at the spectrum maximum is 150 ev in all in this case.

As may be seen from the ion mass-spectrograms (Figure 5) when the plasma is heated by an ion cyclotron wave, there are only hydrogen ions H_1^+, H_2^+, H_3^+ in the plasma. The presence of these ions is apparently due to the fact that the plasma electron temperature is low (20-25 ev). It is particularly interesting to note that all three types of hydrogen ions have approximately the /9 same energy, although the resonance acceleration occurs only for H_1^+.

For purposes of control, the ion energy was measured by another method -- multigrid probe with retarding potential (Ref. 4). This made it possible to record the energy spectrum of electrons. The multigrid probe was placed at a distance of 10 cm from the magnetic mirror. The plasma density was greatly reduced by means of a diaphragm having several 0.1 mm openings. It was possible to separate an electron bundle or an ion bundle, depending on the sign of the pulling voltage on the first grid; the bundles were analyzed by the retarding potential which was supplied to the second grid. In order to decrease the possibility of ionization, a differential pumping was employed to maintain a vacuum of $6.7 \cdot 10^{-4}$ n/m^2 within the probe walls.

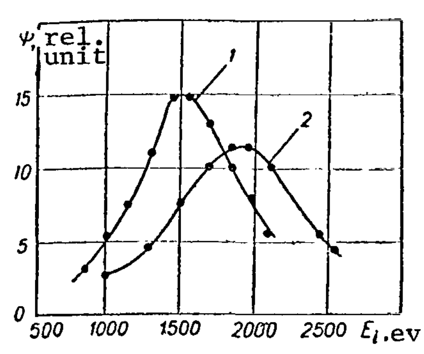

Figure 3

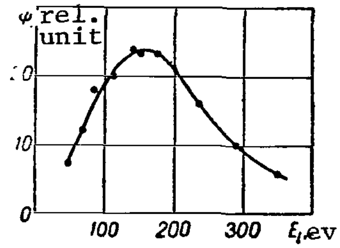

Figure 4

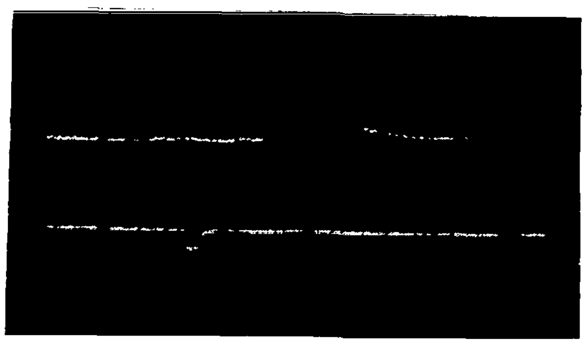

Figure 5

The results derived from measuring the ion energy spectrum by means of the multi-grid probe confirmed the results obtained with analyzer measurements. The proton energy spectrum maximum was in the region of 2 kev when the plasma was heated with ion cyclotron waves. The electron energy remained low, and amounted to 30 ev in all. This result also coincides closely with the results derived from measuring the electron energy by the spectral methods.

As would be expected, the measurements of the energy spectrum of ions and electrons by the multigrid probe, when the plasma was heated with a rapid magnetosound wave, showed that the electron and ion energies were approximately the same (150 ev). The ion energies obtained by measurements with the probe and the analyzer also coincided fairly well.

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MEASUREMENT OF THE PERPENDICULAR ENERGY COMPONENT AND THE PLASMA DECAY TIME DURING HIGH FREQUENCY HEATING /10

N. I. Nazarov, A. I. Yermakov, V. T. Tolok

The correct determination of the charged particle temperature is of paramount importance when studying the effectiveness of plasma heating. One convenient method of determining the plasma temperature consists of measuring its thermal diamagnetism by an external diamagnetic probe which includes a column of the plasma to be heated. The method is based on measuring the difference between the strengths of the magnetic field outside and within the plasma ΔH, which is a function of the gasokinetic pressure. For a plasma with small β $\left(\beta = \frac{8\pi p}{H_0^2}\right)$, this difference is determined by the expression

$$\Delta H = H_0 - H_{within} = \frac{4\pi p}{H_0} \tag{1}$$

where p is the gasokinetic pressure. With a plasma having quasineutrality, we have

$$p = nk(T_{\perp i} + T_{\perp e}), \tag{2}$$

where n is the plasma density; k is the Boltzmann constant. $T_{\perp i}$ and $T_{\perp e}$ are the perpendicular ion and electron temperatures, respectively. Thus, by measuring ΔH and knowing the plasma density, we may compute its temperature.

The temperature was measured by this method on a "Sneg" apparatus

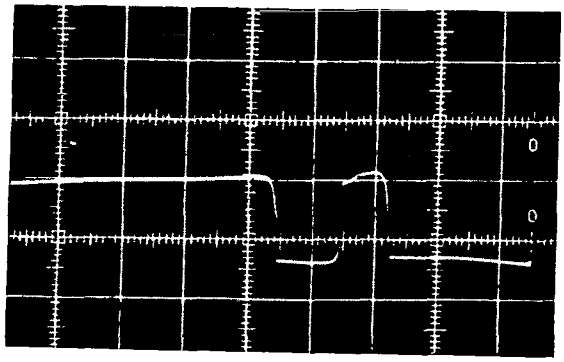

Figure 1

(Ref. 1). Similar measurements were performed in (Ref. 2). The plasma was either produced by a powerful ion cyclotron wave (Ref. 1), or by a rapid magnetosound wave (Ref. 3). Hydrogen in the 0.13-0.4 n/m^2 pressure range was used as the process gas. The waves were excited at a frequency of $f_0 = 10^7$ cps, and the pulse high frequency power, transmitted to the plasma, was 100 kw. In order to avoid transitional processes related to the sharp change in plasma density during the initial period, a coupled, high frequency pulse, whose envelope is shown in Figure 1, was supplied to the exciting coil. The first pulse produced a plasma, and the second pulse was employed to heat it. The duration of the pulses, the amplitude, and the interval between them could be changed independently over very wide intervals.

The period during which the strength of the pulse magnetic field H_0 /11 changed was 24 microseconds. The strength of the field was selected so that either the ion cyclotron wave, or the rapid magnetosound wave, was excited resonantly at the time the second high frequency pulse came into operation. The plasma density was measured by a microwave interferometer at the wavelengths 8.2 and 4 mm.

The quantity ΔH, which was caused by the plasma diamagnetism, was determined by measurements of the electromotive force (emf) by a diamagnetic probe. The probe consisted of two coils, one of which included the plasma column. The other coil was employed to compensate for the emf caused by a change in the strength of the confining magnetic field H_0. In order to eliminate the emf produced by the propagated ion cyclotron wave, a five-unit low-frequency filter was employed, which intersected all frequencies above 3 Mc. The probe was located in the region of the "magnetic beach", at a distance of 30 cm from the edge of the exciting coil. In order to decrease the effect of attenuation of the diamagnetic signal, due to reflection of the charged particles from the walls, the diameter of the discharge chamber was increased up to 8 cm in the region of the diamagnetic probe, while the

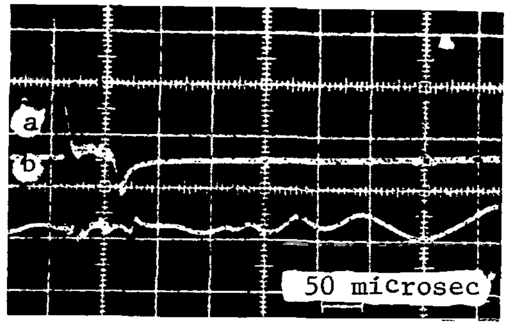

Figure 2

diameter of the plasma with n $> 10^{12}$ cm^{-3} equalled 3.5 cm.

Figure 2 shows an oscillogram of the diamagnetic signal -- a, obtained when the plasma was heated with an ion cyclotron wave; b -- represents the interferogram of an 8-mm signal. The gasokinetic plasma pressure increased very rapidly (in $\sim$10 microseconds), and then barely changed until the end of the high frequency pulse. Since the plasma density changed very little /12 -- (1.2 - 1.5) x 10^{13} cm^{-3} -- during the high frequency pulse, and since it decreased slowly after it had ended ($\tau \sim$ 270 microseconds), it can be assumed that the diamagnetic signal was proportional to the rate at which the perpendicular energy component of the plasma $\frac{d}{dt}(T_{\perp i} + T_{\perp e})$ changed. Figure 3, a shows an oscillogram of a diamagnetic signal. In order to determine the plasma temperature, ΔH was calculated after integration of the diamagnetic signal. The integrated diamagnetic signal is shown in Figure 3, b. The total value of T, thus obtained for resonance excitation of an ion cyclotron wave, amounted to 1 kev. The electron temperature, determined by the spectral method, was in this case 20-30 ev. Thus, the ion temperature was measured indirectly. The plasma temperature was determined by this same method when it was heated by a rapid magnetosound wave. In this case T = 200-300 ev. The values of T, obtained according to the diamagnetic signal, closely coincide with the measured energy of charged particles eminating from the system along the magnetic field (Ref. 4). The small di- /13 vergence between $T_{\perp}$ and $T_{\parallel}$ is caused by the fact that measurements of the plasma temperature according to the diamagnetic signal give an average (over the plasma column cross section) temperature.

Figure 4 shows the dependence of the ion temperature T_i on $\frac{H_0}{H_{ci}}$ (H_0 -- the strength of the outer magnetic field, H_{ci} -- the strength of the magnetic field at which the gyrofrequency of a proton equals 10 Mc). It can be seen that maximum heating occurs with resonance excitation of an ion cyclotron wave.

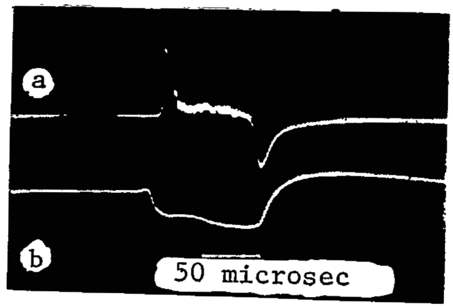

Figure 3

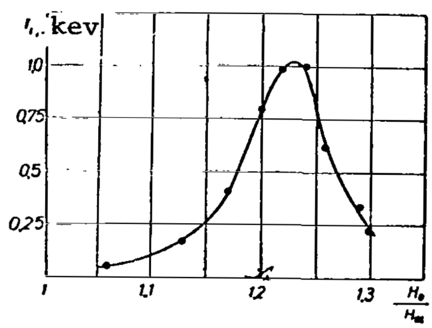

Figure 4

The ion temperature barely changes during heating (see Figure 3). There is a very rapid temperature decrease after the high frequency pulse is recorded. This change in the ion temperature points to the presence of great losses. Since the ion temperature is considerably greater than the electron temperature when this method of plasma heating is employed, it may be assumed that one of the mechanisms for rapid ion cooling is their energy loss when colliding with electrons. However, for a plasma with $n_e \sim 2 \cdot 10^{13}$ and $T_e \sim 25$ ev, the time required to cool hot ions must be 100 microseconds. In actuality, the cooling time equalled 10 microseconds. Therefore, there is no basis for assuming that this is the main loss mechanism.

Another mechanism for rapid ion cooling may be their overcharging, since under the experimental conditions the highly-ionized plasma column was surrounded with a weakly-ionized cold plasma which was in contact with the discharge chamber walls. For a plasma with an electron density on the order of 10^{13} cm^{-3} and a neutral gas pressure at the chamber walls of approximately $1.33 \cdot 10^{-2}$ n/m^2, the probability of overcharging exceeds the ionization probability. Consequently, the losses to overcharging may be considerable, and they continue to increase with an increase in the ion

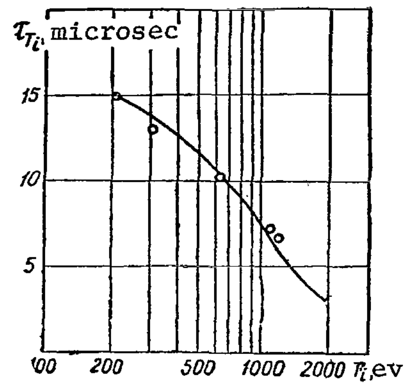

Figure 5

energy. Figure 5 shows the dependence of the ion cooling time on their energy. The solid line corresponds to the time required for overcharging the ions for a density of the surrounding gas of $n_0 = 6 \cdot 10^{12}$ cm^{-3}. This density was selected so that a comparison could be made between the value obtained theoretically and the experimentally measured value τ_{T_i} for T_i = 200 ev. The nature of the dependence of τ_{T_i} on T_i shows that under experimental conditions the energy losses are primarily caused by ion overcharging.

Thus, these experiments enable us to draw the conclusion that a plasma may be heated to a temperature exceeding 1 kev by means of resonance excitation of an ion cyclotron wave. The limiting value T_i is determined by the apparatus parameters. In addition, the results obtained provide a basis for assuming that -- when a hot plasma is insulated from the chamber walls by "vacuum interstratification" -- the time the plasma may be contained may be increased considerably, under the condition that there are no other loss mechanisms. /14

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HIGH FREQUENCY ENERGY ABSORPTION BY A PLASMA IN ION CYCLOTRON RESONANCE IN STRONG, HIGH FREQUENCY FIELDS /15

V. V. Chechkin, M. P. Vasil'yev, L. I. Grigor'yeva, B. I. Smerdov

This article represents a continuation of studies we performed previously (Ref. 1, 2) on high frequency energy absorption by a plasma in ion cyclotron resonance.

As is well known (Ref. 3-5), the heating of a plasma by a variable field at a frequency which is close to the ion cyclotron frequency is very effective, if there are mechanisms leading to energy thermalization of the orderly motion of plasma particles in the field of an ion cyclotron wave. High frequency energy absorption by the plasma may occur, in particular, due to "close" collisions of ions which are in resonance with other types of ions, electrons, and neutral atoms in a cold plasma. This absorption may also be due to "collisionless" cyclotron damping which is caused by the thermal motion of ions in a high temperature plasma. In both cases, it is assumed that the amplitude of the high frequency field is fairly small, so that ions receiving energy in the wave field can transmit it to other particles. If this condition is not fulfilled, nonlinear processes must arise in the plasma, due to which the ion distribution function is essentially distorted. Distortion of the ion distribution function by the ion cyclotron wave with a finite amplitude leads to a decrease in the wave absorption coefficient down to a small value which equals, in order of magnitude, the absorption coefficient for pair collisions at a given temperature.

The expression obtained in the case of a high-temperature plasma (Ref. 5) for a critical field strength of an ion cyclotron wave -- which leads to a significant distortion of the ion distribution function when it is exceeded -- can be written as follows

$$E_{cr} \sim \frac{k_{\parallel} c}{\omega} \cdot \frac{H}{(\omega\tau)^{2/3}} \left(\frac{8\pi N_i T_i}{H^2}\right)^{1/3} \quad (1)$$

where ω is the wave frequency; $k_{\parallel} = \frac{2\pi}{\lambda}$; λ -- axial wave length; H -- constant magnetic field; τ -- relaxation time of ions due to ion-ion collisions; N_i -- ion density; T_i -- ion temperature. /16

In the case of a cold plasma, the critical strength of the wave electric field (at the absorption maximum) is as follows

$$E_{cr} \sim \frac{M}{e\gamma_{eff}} \sqrt{\frac{T_i}{M}}, \tag{2}$$

When this value is reached, the velocity of the ordered motion of an ion liquid, with respect to electrons, equals the thermal ion velocity. M -- ion mass; e -- ion charge; γ_{eff} -- effective frequency of ion collisions.

If the velocity of the relative motion of ions and electrons in the field of an ion cyclotron wave is greater than the ion thermal velocity, "bunched" instability may arise in the plasma, which is related to the excitation of high frequency (as compared with $\omega_i = \frac{eH}{Mc}$) longitudinal fluctuations, whose increasing increment is considerably greater than the cyclotron ion frequency, and the wavelength is considerably less than the ion cyclotron wavelength. The excitation of these fluctuations by an ion bundle moving in a direction which is perpendicular to a constant magnetic field has been investigated in (Ref. 6-8).

Similar small-scale electrostatic plasma fluctuations, which take place in a wide frequency and wave number range, must lead to an increased exchange of energy between plasma ions and electrons (as compared with the exchange caused only by Coulomb collisions), and must also lead to a significant increase in all the transfer coefficients across a constant magnetic field.

Let us examine certain results of experimental studies in this light (Ref. 9, 10). In these studies, the field strength of an ion cyclotron wave exceeds by at least one order of magnitude the critical field strength (1), and the time of ion-ion relaxation is either a comparable with the plasma decay time (Ref. 9) or considerably exceeds it (Ref. 10). For this reason, the strong high frequency energy absorption by the plasma and the ion heating, observed in these studies, cannot -- in our opinion -- be caused by cyclotron damping. In addition, the study (Ref. 11), which was carried out on the same apparatus as was employed in (Ref. 10), observed a rapid plasma decay if the high frequency power introduced into the plasma exceeded a certain critical value. This rapid decay was apparently caused /17
by "bunched" instability.

The phenomenon of anomalous plasma diffusion across a magnetic field, produced for a critical value of a high frequency field strength with a frequency close to ion cyclotron frequency, was discovered and studied in detail in (Ref. 2). In particular, this study found that increased diffusion occurs if the velocity, acquired by ions in an azimuthal high frequency field during the period between collisions, is comparable to the thermal ion velocity or exceeds it -- i.e., if relationship (2) is fulfilled.

The study (Ref. 12) performed an experimental determination of the increase in the effective frequency of ion collisions in a low-density plasma under conditions of ion cyclotron resonance in strong, electric, variable fields. It was found that the dependence obtained experimentally for the frequency of ion collisions on the high frequency field strength cannot be explained on the basis of a theory postulating ion collisions with neutrals. The assumption was advanced that such a dependence is caused by "bunched" instability which arises in strong variable fields.

We can clarify the nature of this absorption by comparing the data obtained experimentally, regarding high frequency energy absorption by a cold plasma close to ion cyclotron resonance, with the theory advanced in (Ref. 5). Let us establish a relationship between the effective frequency of ion collisions and anomalous plasma diffusion in ion cyclotron resonance, which was studied in (Ref. 2). Finally, by making certain numerical estimates, we can show that both high frequency energy absorption by the plasma, and anomalous plasma diffusion in fields with supercritical strength values, may be caused by "bunched" instability produced in the field of an ion cyclotron wave (this instability was investigated theoretically in [Ref. 8]).

Description of the Apparatus. Measurement Methods

The apparatus which was employed for the study was described in detail in (Ref.1). The plasma was produced by pulse discharge with oscillating electrons in hydrogen, in a $7 \cdot 10^{-2}$ — 7 n/m^2 pressure range. The diameter of the glass discharge tube was 6 cm, and the distance between /18
the cathodes was 80 cm. The strength of the longitudinal, quasi-constant magnetic field could be changed between $4 \cdot 10^4$—$6.4 \cdot 10^5$ a/m.

High frequency energy was introduced into the plasma by means of an artificial LC-line which was connected to the self-excited oscillator with a frequency of $7.45 \cdot 10^6$ cps. During excitation at this frequency, 2.5 wavelength oscillation was applied to the section of the line which was slipped on to the discharge tube. This corresponded to an axial high frequency field period of 23 cm. By changing the anode strength of the oscillator, and also the connection between the line and the oscillator, it was possible to change the amplitude of the high frequency azimuthal line current within 0.5 - 35 a/cm.

The oscillator was switched on for approximately 100 microseconds after the pulse of the discharge current had terminated. As is shown in (Ref. 1), the maximum high frequency power which could be absorbed by a plasma in resonance was 18 kw for a plasma density of $1.7 \cdot 10^{13}$ cm^{-3}, and an azimuthal line current of 30 a/cm.

This article presents the measurements of plasma density, electron temperature, and the high frequency power absorbed by the plasma. The

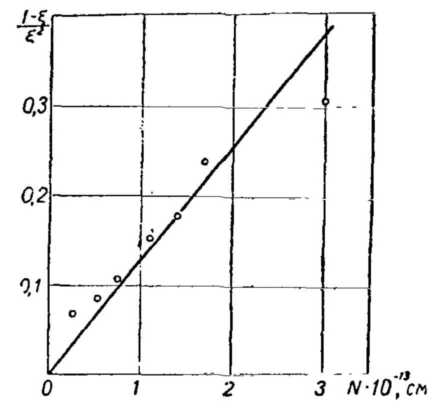

Figure 1

electron plasma density and its change with time in the (1.7 - 0.25) 10^{13} cm^{-3} range was measured by means of an interferometer at a wavelength of 8.1 mm = 0.81 cm. The experimental point corresponding to a density of $3 \cdot 10^{13}$ cm^{-3} was obtained by extrapolation of the plasma density dependence on time toward large densities (Figure 1).

The plasma electron temperature was determined according to the dependence of the luminosity intensity of the line H_β on the high frequency LC-line current [see (Ref. 2)]. The luminosity intensity of the line H_β was measured by means of a UM-2 monochromator and a photoelectron multiplier. Due to anomalous diffusion, the plasma decay time, for line currents on the order of 5 a/cm and above, was comparable with the electron lifetime between two collisions leading to excitation of a neutral atom. /19 For this reason, for line currents which were greater than, or approximately equal to, 5 a/cm, the electron temperature which was computed according to the line intensity H_β could be too low.

The high frequency power absorbed by the plasma was measured with an all purpose meter of transmitted power which was described in (Ref. 13). The recorders for the current and the strength were connected to the line at the point where it was attached to the solenoid. By means of two interchangeable current recorders with different sensitivity, it was possible to measure the power absorbed by the plasma, corresponding to a line current of 0.5 - 10 a/cm.

All of the measurements described in this article were performed at a hydrogen pressure of 0.133 n/m^2.

Nature of High Frequency Energy Absorption

Under the conditions of our experiment, the high frequency power absorbed per unit of plasma cylinder length is (Ref. 5)

$$S = \frac{\pi^2 R^2}{4ca} K_1^2 (k_\parallel R) \left(\frac{a\omega}{c}\right)^5 n_\parallel^4 j_0^2 f(X) \sin^2 k_\parallel z, \quad (3)$$

where j_0 is the amplitude of the azimuthal high frequency current in the coil per unit of cylinder length; R -- coil radius; a -- plasma radius (it is assumed to equal the inner radius of the discharge tube in all of the computations); K_1 -- McDonald function; $k_\parallel = \frac{2\pi}{\lambda}$ -- axial wave number (it is determined as the axial period λ of high frequency current in the coil); ω -- oscillator frequency; $n_\parallel = \frac{k_\parallel c}{\omega}$; f(X) -- the function whose specific form depends on the nature of the high frequency energy absorption. In particular, for collision absorption we have

$$f(X) = \frac{X \gamma_{eff} \omega_i}{(X\omega_i + \omega - \omega_i)^2 + \gamma^2_{eff}}, \quad (4)$$

where

$$X = \frac{n_A^2}{2n_\parallel^2};$$

$$n_A^2 = \frac{4\pi N_i M c^2}{H^2}.$$

Strictly speaking, relationship (3) is only valid in the case of long-wave fluctuations (or a small plasma filament radius), when $k_\parallel a \ll 1$ and $k_\perp a \ll 1$ ($k_\perp$ -- radial wave number). Under our conditions, both of these quantities are on the order of unity. However, as the computation showed, the error produced when equation (3) is applied to our case is small. /20

Under the conditions of the described experiment, high frequency energy absorption by a plasma close to $\omega = \omega_i$ is "collision absorption" in the sense that it is described by (3), where f(X) has the form of (4). Formulas (3) and (4) may be used to determine the fact that, in the case of a hydrogen plasma, the absorbed power is at a maximum in the case of

$$\frac{1-\xi}{\xi^2} = 2{,}44 \cdot 10^{-17} \lambda^2 N_i, \quad (5)$$

where $\xi = \frac{H_0}{H_m}$; H_0 is the magnetic field corresponding to cyclotron

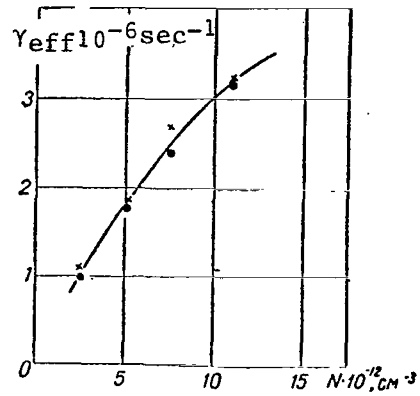

Figure 2

resonance of ions at the oscillator frequency; H_m -- the magnetic field forwhich the high frequency power absorbed by the plasma is at a maximum.

The dependence described by (5) is shown in Figure 1 (straight line). This figure also presents the points computed on the basis of experimental data regarding the shift in the absorption maximum from H_0 for different plasma densities measured by an interferometer for a line current of 4.5 a/cm. Over the entire range of measured plasma densities, the deviation of the points obtained experimentally from the computed dependence does not exceed the limits of measurement errors.

In order to determine what collisions cause the observed high frequency energy absorption, a study was made of the dependence of the effective frequency of ion collisions on the neutral gas pressure and on the plasma electron density. It can be seen from (3) and (4) that the effective frequency of ion collisions γ_{eff} can be expressed either as $\frac{1}{2} \cdot \frac{e}{Mc} \Delta H$ -- where ΔH is the halfwidth of the resonance absorption curve -- or as $\frac{AX\omega_i}{S_m} j_0^2$, where A is the factor in front of $j_0^2 f(X)$ in (3); S_m -- the power at the absorption maximum.

It was shown in (Ref. 1) that the power at the absorption maximum and the width of the resonance absorption curve depend slightly on the /21 neutral gas pressure in the 0.133 - 1.33 n/m^2 range, due to which fact the observed power absorption cannot be caused by ion collisions with neutrals.

Figure 2 presents the dependence obtained experimentally of the effective frequency of ion collisions γ_{eff} on the plasma electron density for a line current 4.5 a/cm. The crosses designate the points computed

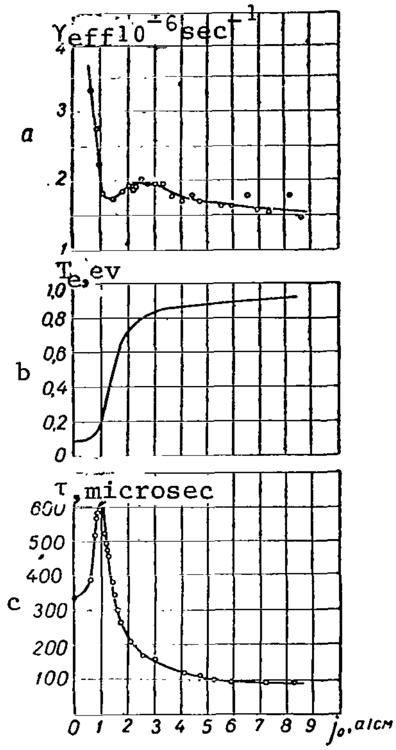

Figure 3

according to the halfwidth of the absorption curves; the dots designate the points computed according to the absolute power at the absorption maximum for a given density. The observed high frequency energy absorption is caused by ion-electron collisions. However, under the conditions for which the dependence shown in Figure 2 was determined, the electron temperature comprised 1 ev in order of magnitude (see Figure 3). The frequency of ion Coulomb collisions with electrons must be one order of magnitude less than γ_{eff} over the entire range of measured plasma densities. Therefore, the effective frequency of ion collisions, which was measured in the experiment, cannot be caused by Coulomb scattering of ions by electrons [the electron temperature, which was determined in (Ref. 1) according to γ_{eff} under the assumption of Coulomb interaction of ions with electrons, amounted to 0.15 ev). /22

In order to clarify the nature of the interaction between ions and electrons, and consequently the nature of the observed high frequency energy absorption by a plasma in ion cyclotron resonance, the dependence of the effective frequency of ion collisions on the line current density j_0 -- i.e., the dependence on the strength of the high frequency field in the plasma -- was determined. The quantity γ_{eff} was computed according to the power at the absorption maximum for a density of $5.1 \cdot 10^{12}$ cm^{-3}. Figure 3 shows the dependences γ_{eff} (j_0), $T_e(j_0)$ and $\tau(j_0)$ -- where τ is the plasma decay time -- on density $7.6 \cdot 10^{12}$ up to $2.5 \cdot 10^{12}$ cm^{-3} for a magnetic field strength of $4.2 \cdot 10^5$ a/m, where the absorption reaches a maximum for a density of $5.1 \cdot 10^{12}$ cm^{-3}. It can be seen that γ_{eff} rapidly decreases with an increase in $j_0 (j_0 < 1$ a/cm). The frequency of Coulomb collisions between ions and electrons must follow this pattern, since the high frequency power introduced into the plasma -- and consequently the electron temperature -- increases with an increase in j_0. The electron temperature, determined according to γ_{eff} in the case of $j_0 < 1$ a/cm under the assumption of Coulomb interaction between ions and electrons, coincides in order of magnitude with the temperature computed according to the line intensity H_β (see Figure 3).

There is a sharp bend in the curve γ_{eff} (j_0) at the point $j_0 \leq 1$ a/cm, and with a further increase in j_0, γ_{eff} increases slowly, remaining at a level on the order of $2 \cdot 10^6$ sec^{-1} and considerably exceeding the frequency of Coulomb collisions between ions and electrons for given electron temperatures. As can be seen from Figure 3, at this point a sharp decrease in the plasma decay time occurs. It was shown in (Ref. 2) that a decrease in the decay time is caused by increased plasma diffusion across the force lines of the magnetic field. In its turn, the diffusion is caused by an instability produced in the field of an ion cyclotron wave having a large amplitude.

It may thus be assumed that under the conditions of our experiment the anomalously large frequency of ion collisions in high frequency fields with supercritical strengths is caused by the more intense (as compared with Coulomb interaction) interaction of ions with electrons. The reason for this (and for anomalous diffusion) is "bunched" instability. The fact that absorption is "collision absorption" -- i.e., it formally satisfies equation (3) -- where f(x) in the form of (4) means in physical terms that the damping force (which is caused by an instability) of the ion motion /23
directed toward the electrons is proportional to the relative velocity of ion and electron liquids in the field of an ion cyclotron wave.

Comparison With the Theory of Ion Cyclotron Wave Stability

We shall show that the values of the effective frequency of ion collisions in fields with supercritical strengths, computed either according to the halfwidth of the resonance absorption curve or according to the absolute power at the absorption maximum, as well as the plasma diffusion coefficient determined by the decay time, coincide in order of magnitude with the corresponding values computed for our case according to the theory of ion cyclotron wave stability (Ref. 8).

As follows from (Ref. 8), under the experimental conditions ($N_e \sim \sim 5 \cdot 10^{12}$ cm^{-3}; $H \sim 4 \cdot 10^5$ a/m; $T_e \sim 1$ ev) for longitudinal (electrostatic) high frequency fluctuations excited by an ion bundle and propagated almost across a constant magnetic field ($\cos^2 \Theta \lesssim \frac{m}{M}$), we obtain

$$\mathrm{Re}\,\omega \sim \mathrm{Im}\,\omega \sim (\omega_e \omega_i)^{1/2}, \tag{6}$$

where ω_e is the cyclotron electron frequency.

The effective frequency of ion collisions may be regarded as the inverse of the time during which an ion bundle with an initial energy per unit of volume $N_i \frac{Mu^2}{2}$ (u -- the velocity of the relative motion of the ion and electron components in the field of an ion cyclotron wave almost equals the velocity of an ion liquid) excites the fluctuations (6) and transmits all of the energy of the ordered motion to the electron gas. On the other hand, the electrons obtain the energy $Ni \frac{mu^2}{2}$ from the ions during the excitation time of the fluctuations $(\omega_e \omega_i)^{-1/2}$. The braking time of an ion bundle is thus

$$\tau = \frac{M}{m} (\omega_e \omega_i)^{-\frac{1}{2}}. \tag{7}$$

Substituting numerical values in (7), we obtain 10^{-6} sec for τ, which coincides in order of magnitude with $\gamma_{eff}^{-1} \sim 0.5 \cdot 10^{-6}$ sec which was determined experimentally.

Let us determine the diffusion caused by instability, employing the theory of nonuniform plasma stability (Ref. 14). The diffusion coefficient may be written as follows /24

$$D \sim V^2 t, \tag{8}$$

where V is the plasma pulsation velocity; $t = \frac{1}{\nu}$ -- the characteristic time of correlation disappearance. The increasing increment of fluctuations (6) must be used as ν. The pulsation amplitude may be determined by the condition of balance between two processes -- one of which leads to an increase in the pulsation amplitude due to the development of instability, and the other leads to contraction of this amplitude due to nonlinear processes leading to oscillation damping. As a result, we obtain

$$V = \nu \lambda_{\perp}, \tag{9}$$

where $\lambda_{\perp}$ is the oscillation wavelength in the radial direction. Then the diffusion coefficient is

$$D \sim \nu\lambda_{\perp}^2, \tag{10}$$

or, since ku $\approx \nu$, where k is the radial wave number, we have

$$D \sim (2\pi)^2 \frac{u^2}{\nu}. \tag{11}$$

Under the experimental conditions, in the case of j_0 = 8 a/cm, u ~ ~ 10^6 cm/sec and D ~ $2 \cdot 10^4$ cm^2/sec, which coincides in order of magnitude with D ~ $5 \cdot 10^4$ cm^2/sec, determined for the same conditions according to the plasma decay time in (Ref. 2).

Conclusions

1. The absorption of high frequency energy, which was observed in our experiments, close to the ion cyclotron resonance both for small and large amplitudes of the high frequency field can be formally described by relationships (3) and (4), which were derived for the case of "collision" absorption.

2. Absorption is caused by the interaction between plasma ions and electrons. In fields whose strength is less than the critical strength, it is caused by pair Coulomb collisions. In fields whose strength is greater than the critical strength, the effective frequency measured experimentally of ion collisions is considerably greater than the frequency of pair Coulomb collisions. In this case, the anomalously large /25 absorption is apparently caused by braking of the ordered motion of ions in the field of the ion cyclotron wave by high frequency longitudinal oscillations excited by an ion bundle.

3. The value obtained experimentally for the effective frequency of ion collisions, and also the diffusion coefficient determined according to the plasma decay time, coincide in order of magnitude with the corresponding values calculated according to the theory of ion cyclotron wave stability.

4. On the basis of the results obtained, it is natural to pose the question of "turbulent" plasma heating (Ref. 15) in ion cyclotron resonance -- i.e., brief excitation in the plasma of an ion cyclotron wave having a large amplitude, with subsequent thermalization of the ordered motion of plasma particles in the field of this wave by the high frequency longitudinal oscillations excited by an ion bundle passing through the electron gas. The heating period must be quite small, so that anomalous diffusion caused by plasma instability does not produce significant losses in plasma particles.

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INVESTIGATION OF CONDITIONS PRODUCING A DENSE PLASMA IN A METALLIC CHAMBER AND ITS HIGH FREQUENCY HEATING

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It was shown in (Ref. 1) that powerful, high frequency oscillators may be employed to produce a dense plasma in a metallic chamber. The present experiments represent a continuation of this study by investigating the conditions producing a dense plasma in a metallic chamber and by determining the main plasma parameters.

The utilization of a metallic chamber as the operational vacuum body and the method of feeding the system from the hf oscillator have definite advantages and unusual features.

1. The system has a low input impedance, which does not require high voltages when large, high frequency powers are introduced, and avoids several technical difficulties which accompany, as an example, the spatially periodic circuit proposed by Stix (Ref. 2).

2. A good connection between the feed electrodes and the plasma is provided. A dense plasma with a cylindrical form is produced between the central electrodes, and has no direct contact with the wall of the discharge chamber.

Investigation of the Conditions Producing a Dense Plasma in a Metallic Chamber

The "Vikhr'" device (Figure 1), on which the study was performed, consists of a copper chamber 1 with a wall thickness of 2.5 mm, an inner diameter of 125 mm, and a length of 2000 mm. It is placed in a magnetic field which /27
can be regulated continuously between $0 - 2 \cdot 10^5$ a/m. The configuration of the magnetic field may be varied, depending upon the method chosen to handle the plasma produced. Since we were interested in plasma heating by high

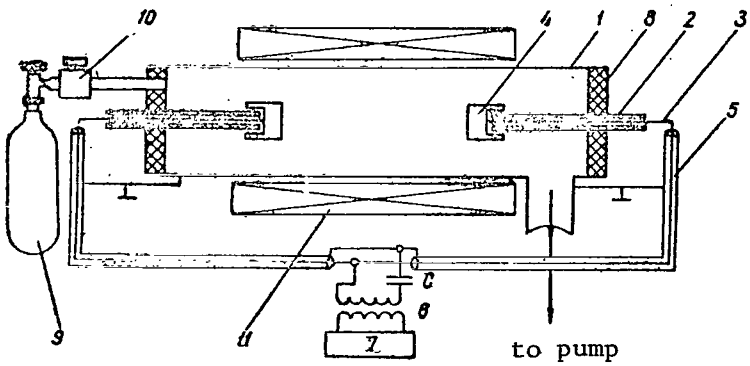

Figure 1

frequency fields, the magnetic field configuration was chosen in accordance with the conditions which were requisite for generation and absorption of ion cyclotron waves. The region of the "magnetic beach" was located in the center of the selenoid 11 producing the magnetic field. The residual gas pressure in the system did not exceed $1.33 \cdot 10^{-4}$ n/m^2. The rods 3 are introduced axially into the vacuum chamber through the porcelain insulators 2. Aluminum electrodes 4 with a diameter of 50 mm and a length of 70 mm are located at the ends of these rods. The distance between the electrodes is 1000 mm. The feed rods 3 through the coaxial cables 5 and the capacitance C are connected with the coupling coil 6 of the hf oscillator 7. The oscillator power is on the order of 100 kw; the operating frequency is $1.82 \cdot 10^6$ cps. The ends of the chamber are closed with glass discs 8. The operational gas from the flask 9 is admitted into the chamber by the valve 10 through an opening in the glass disc 8.

With the system employed for switching on the hf oscillator, the feed electrodes acquire a negative potential, due to the rectifying properties of the plasma; this negative potential can reach several kilovolts with respect to the chamber. This potential creates the condition for oscillation of electrons along the magnetic field force lines between electrodes, similarly to Penning discharge. The oscillating electrons effectively /28
ionize the operational gas and provide for a high degree of ionization up to a chamber pressure of $1.33 \cdot 10^{-2}$ n/m^2. A core of dense plasma is formed between the electrodes along the chamber axis; the diameter of this plasma is determined by the electrode diameter. Due to the presence of a constant potential, the peripheral plasma begins to rotate according to the law of plasma behavior in crossed electric fields and magnetic fields. In a few microseconds the applied high frequency voltage produces a discharge in /30
the operational area, which is accompanied by a voltage decrease at the electrodes due to an increase in the oscillator loading (Figure 2, a). The central electrode acquires a negative potential, which is retained for the entire period of time that the dense plasma exists (Figure 2, b).

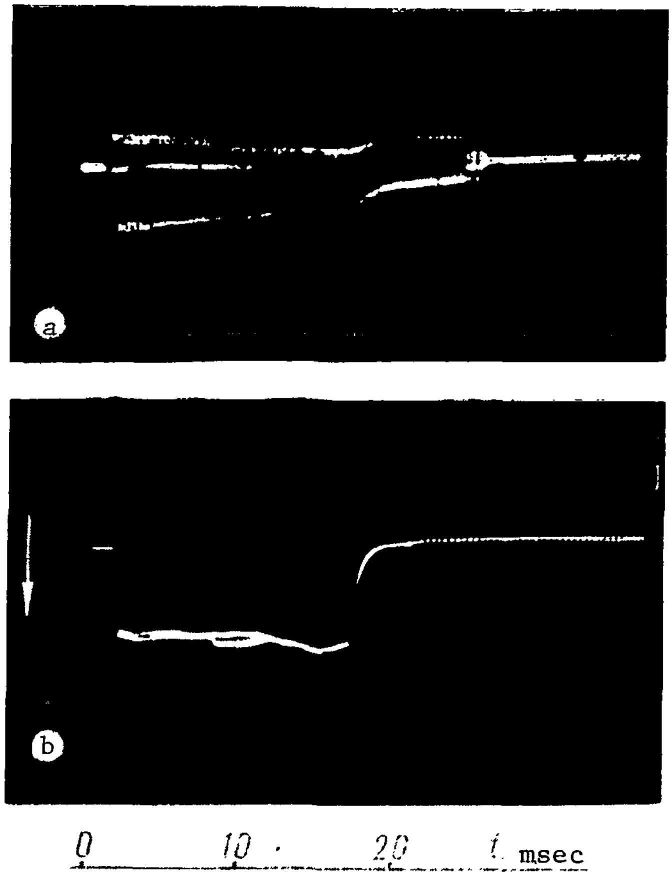

Figure 2

Lines of admixtures from the chamber wall materials and from the electrodes -- CuII, AIII -- and a weak line CII (Figure 3) were observed in the discharge spectrum obtained by means of a ISP-51 spectrograph with a UF-85 camera (focal distance 1300 mm).

When the outline of the line H_β was measured, it was found that under our experimental conditions there was an apparant Stark widening by the micropoles of the plasma. This made it possible to determine the charged particle density, by comparing the contour observed experimentally with the theoretical contour computed on the basis of the theory advanced by Kolb, Grim, and Shen (Ref. 3). For purposes of comparison, Figure 4 shows the observed contour I and the theoretical Stark contour II, computed for $n = 2.0 \cdot 10^{14}$ cm^{-3}. The figure also plots the Gaussian contour III with the halfwidth equalling the experimental value of the halfwidth 0.7 Å. Close to the maximum, the line broadening was caused by the Doppler mechanism, and the slopes of the line are due to broadening by the Stark

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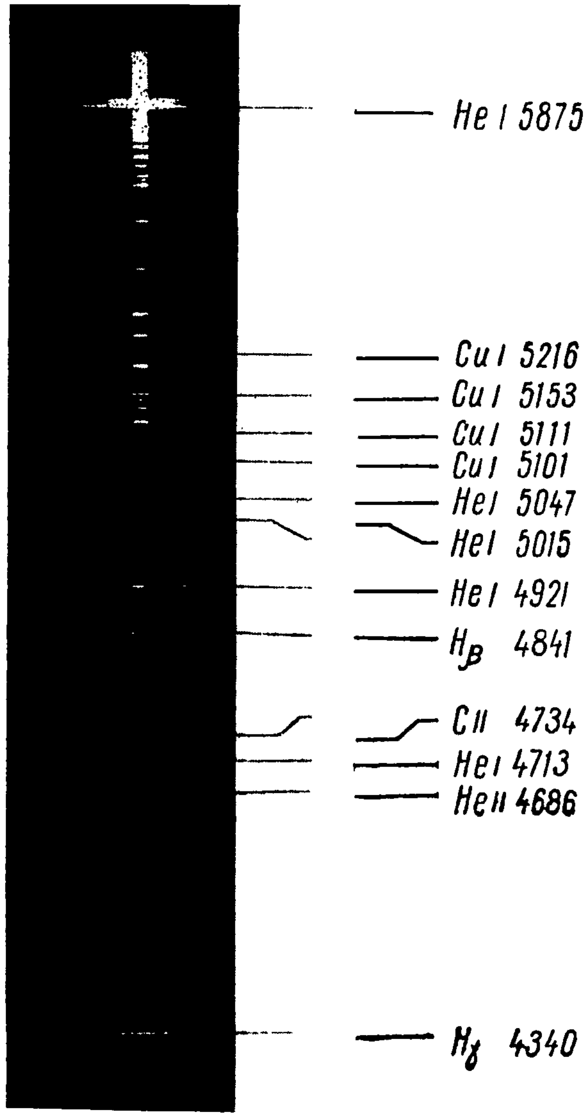

Figure 3

mechanism.

Measurements of the plasma density made it possible to determine its comparatively weak dependence in discharge on the magnetic field strength, which confirmed the assumption regarding the nonresonance mechanism by which a plasma is produced in discharge. At a pressure of 0.4 n/m^2, the maximum plasma density was $2 \cdot 10^{14} \frac{1}{cm^3}$. For an approximate determination of

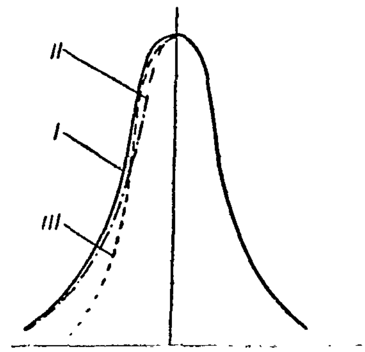

Figure 4

the plasma density distribution over the radius of the system, different sections of the plasma were focused on the spectrograph slit. The most dense plasma (density on the order of $10^{14} \frac{1}{cm^3}$) produces a filament with a diameter of 20 mm; at a distance of about 30 mm from the discharge axis, the plasma density decreased by more than one order of magnitude.

The electron temperature was measured with respect to the intensities /32
of singlet and triplet lines of helium; it was (4 - 5) 10^5 °K.

The dependence of the plasma density change on time was determined by probing the plasma with an ultrahigh frequency signal at a wavelength of 3 cm (Figure 5, c) and 0.8 cm (Figure 5, d) through the glass openings in the chamber. The measurements showed that in an optimum regime there is a plasma with a density of $\geq 10^{13} \frac{1}{cm^3}$ in the apparatus for 3.6 milliseconds, and with a density of $\geq 10^{12} \frac{1}{cm^3}$ for 17 milliseconds. The form of the hf pulse is shown in Figure 5, a. The maximum plasma density was determined according to measurements of the relative change in the intensity of the spectral line H_β with time. The intensity of the spectral line H_β (the photomultiplier signal, Figure 5, b), if the main excitation mechanism is electron collision, is

$$I_{42} = \frac{h\nu_{42} A_{42} n_a n_e \langle v\sigma_4(v) \rangle}{A_{42} + A_{41}},$$

where ν_{42} is the frequency corresponding to the line H_β; A_{ik} -- probability of spontaneous transition from the i level to the k level;

$$\langle v\sigma_4(v) \rangle = \int_0^\infty v\sigma_4(v) f_e(v)\, dv;$$

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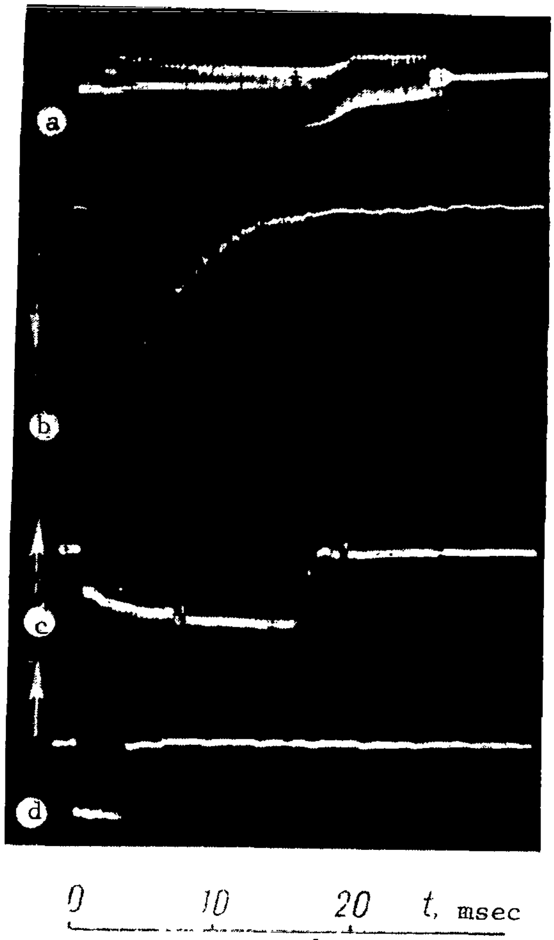

Figure 5

$\sigma_4(v)$ -- excitation cross section (by electron collision) of the n = 4 level. $f_e(v)$ is the electron distribution function with respect to velocity; $[v\sigma_4(v)]$ in the case of Maxwell distribution of electron velocities is the function of electron temperature which -- as measurements of the dependence $T_e(t)$ have shown -- changes very little during the duration of the hf pulse. Thus, the intensity of the line H_β must change proportionally to the product $n_a n_e$. In our case, the neutral gas from the cold section constantly enters the discharge column, since $\frac{R_{pl}}{v_n} \approx 10^{-5}$ sec (R_{pl} -- radius of the plasma column; v_n -- thermal velocity

of the neutrals). Therefore, the neutral density $n_a \approx$ const and, consequently, the intensity of the line H_β is proportional to the electron density n_e. At the moment when $n = 10^{12} \frac{1}{cm^3}$, the intensity of the line H_β decreases by a factor of 200 as compared with its maximum intensity -- i.e., the maximum plasma density in our case is $2 \cdot 10^{14} \frac{1}{cm^3}$, which coincides with the contour of the line H_β determined according to Stark broadening. The plasma density determination based on the intensity of the line H_β, at the moment when a signal with a wavelength of 0.8 cm /33
begins to pass through, coincides with the result of microwave measurements.

Thus, we have studied the conditions producing a dense plasma in a metallic chamber. A plasma with a density on the order of $10^{14} \frac{1}{cm^3}$ and an electron temperature (4 - 5) 10^5 °k was obtained in the experiments. The weak dependence of plasma density on the magnetic field strength points to a nonresonance mechanism by which the plasma is produced. The aim of our subsequent experiments was to investigate the heating of the plasma obtained by the generation of ion cyclotron waves.

High Frequency Heating of a Dense Plasma in a Metallic Chamber

The next stage of our investigation was to study the possible heating of ions by generating ion cyclotron waves in a dense plasma produced by a hf oscillator in a metallic discharge chamber. The "Vikhr'" device was modernized so that, when the ion cyclotron wave was generated at the ends of the coaxial close to the magnetic mirror, it was propagated into the center of the discharge chamber, where the magnetic field strength was decreased to a value equalling the cyclotron value for protons, forming the region of the magnetic beach. When the wave was propagated along the system axis and approached the region of the magnetic beach, its velocity decreased, and it was damped, transmitting its energy to the plasma ions. In order to propagate the wave in the region of the magnetic beach, it is necessary that the magnetic field strength per wavelength change by several percents -- i.e., a smooth change in the magnetic field strength over the length of the system is requisite. If the opposite is true, the wave will be reflected.

The residual gas pressure in the system did not exceed $1.3 \cdot 10^{-4}$ n/m^2. The experiment was performed with a mixture of two gases -- hydrogen and helium or hydrogen and argon. The operational pressure was established by a stationary regime of the valve operation in the 0.1 - 0.8 n/m^2 range. At a pressure of 0.4 n/m^2, the hf oscillator, operating at a frequency of $1.82 \cdot 10^6$ cps with a power of 150 kw, produces a plasma with a density on the order of $10^{14} \frac{1}{cm^3}$. With a magnetic field strength of $H = 1.25\, H_{i.c}$ ($H_{i.c}$ -- the cyclotron value of the magnetic field strength for protons) waves are generated in the plasma with $\lambda = \frac{\lambda_{vac}}{n} = 6.6$ cm /34 (λ_{vac} -- wave length in a vacuum; $n = \frac{c}{v_A \sqrt{1 - \left(\frac{\omega}{\omega_{i.c}}\right)^2}}$ -- refractive index of the medium; $v_A = \frac{H}{\sqrt{4\pi\rho}}$ -- Alfven velocity; H -- magnetic field; ω -- operational frequency of the oscillator; $\omega_{i.c}$ -- cyclotron frequency of proton rotation; ρ -- mass density.

This system has several advantages. The pulse which is initially transmitted to the ions is perpendicular to the vector of the outer magnetic field strength, which facilitates the retention of ions in the corkscrew configuration of the magnetic field. Since the system has a low inpedance, energy is readily introduced into the discharge chamber and is transmitted directly to the plasma ions. An increase in the density and diameter of the plasma does not make the conditions worse for wave generation. At the same time, the spatially periodic circuit for introducing hf power into the plasma, which was advanced by Stix (Ref. 2), loses any physical meaning with an increase in the plasma density and diameter. When a mixture of two gases is heated, or when it is necessary to heat the plasma electron component simultaneously with the ion component, this system makes it possible to introduce the power of two oscillators operating at different frequencies.

As has been pointed out, an increase in the plasma density does not impede wave generation, since spatial periodicity is not given externally, but is established as a function of the plasma refractive index, and may be small (several centimeters). However, the possibility of periodicity is not excluded, if rings are placed endwise on the coaxial at different distances from each other; these rings will introduce a perturbation, creating a specific periodicity along the system axis. One unusual feature of the discharge is the fact that its nucleus, consisting of a plasma which is almost entirely ionized, is surrounded by a plasma having a low density and a neutral "housing". During generation and absorption of ion cyclotron waves, the dense plasma nucleus is heated, and the cold plasma surrounding it with a low density contributes, in all probability, to the suppression of channel instability.

In the experiments described, the ion temperature of the plasma and the charged particle density, which were averaged over time, were determined by means of optical methods. The change with time in the intensity of the spectral lines for hydrogen and admixtures was also studied. A ISP-51 spectrograph was placed in such a way that the central portion of /35

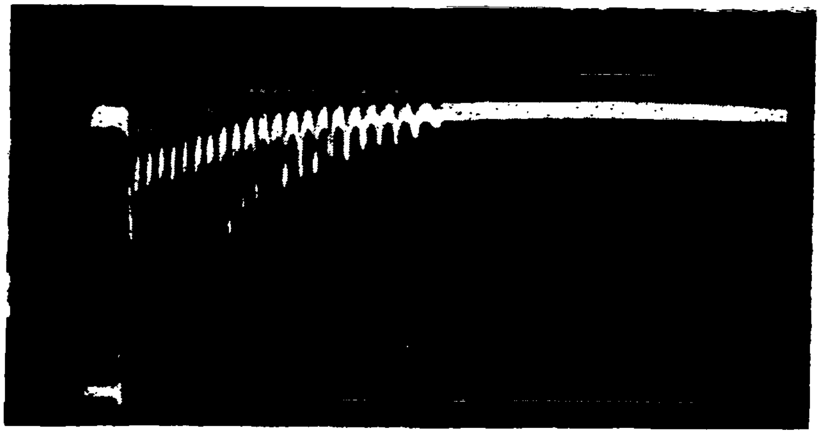

Figure 6

the cylindrical plasma column, lying in the region of the magnetic beach, was focused on its input slit. Oscillograms showing the intensity of the spectral lines for hydrogen and helium or argon, added to the chamber in small proportions to the operational gas, showed that regular oscillations are produced in the intensity of the spectral lines with a frequency on the order of 20 kc, close to the magnetic field strength corresponding to the cyclotron value for protons (Figure 6). These oscillations appear during wave generation in the plasma, and are caused by the eccentric rotation of the dense plasma filament as a whole with respect to the chamber axis, in accordance with the drift law in crossed, radial electric fields and axial magnetic fields. The direction and frequency of the filament rotation was determined by two photoelectron multipliers oriented towards the end of the chamber and placed at a radius of 3 cm from the system axis. One of them was shifted along the azimuth. The oscillation phase of the light intensity was thus changed.

One interesting feature was discovered when the ion temperature was measured according to the Doppler broadening of the hydrogen and additional gas lines. The width of the hydrogen line depended comparatively little on the magnetic field strength, and the additional gas lines were broadened considerably when the magnetic field strength was close to the cyclotron value for protons. The ion temperature of this gas, determined for the optimum operational regime of the apparatus, amounted to 250 ev ($2.5 \cdot 10^6$ °K). /36
Measurements of the halfwidth of the line H_β showed that the hydrogen atom temperature was below the temperature of the additional gas, while there was considerable Stark broadening of the line contour, corresponding to a plasma density of $2 \cdot 10^{14} \frac{1}{cm^3}$. There was also Doppler broadening of the admixture lines (copper, aluminum, oxygen, carbon, nitrogen). When the temperatures of different admixtures and additional gases were measured,

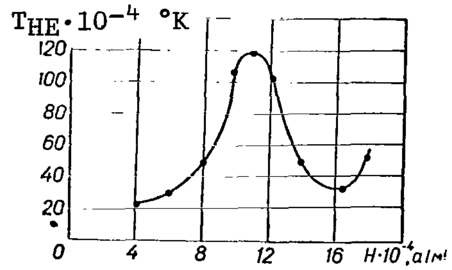

Figure 7

no ion temperature dependence on their mass was detected. The heating of the additional gas ions was terminated when only one additional gas was admitted into the discharge chamber up to the previous operational pressure. One of the characteristic dependences of helium temperature (when it was added to the chamber in a small amount) on magnetic field strength, measured over the halfwidth of a helium line with a wavelength of 4921.93 Å, is shown in Figure 7. The helium atoms acquire a maximum temperature at a magnetic field strength which is close to the cyclotron value for protons. A certain temperature increase is observed as a magnetic field strength is approached which equals the double cyclotron value for hydrogen ions.

If it is assumed that the halfwidth of the H_β line is determined by Doppler broadening, the temperature of neutral hydrogen is considerably lower than the plasma ion temperature. However, under our conditions (in the case of $T_e = [4 - 5] \cdot 10^5$ °K), the lifetime of neutral hydrogen in the plasma, with respect to the ionization process, was small as compared with the time of Coulomb collisions. Therefore, neutral hydrogen cannot acquire energy equalling the ion energy. Since the frequency of hydrogen atom collisions with electrons is greater under these conditions than the frequency of collisions with ions, the Stark mechanism is the predominant mechanism leading to the broadening of the line of the residual neutral hydrogen.

In an optimum operational regime in a mixture of two gases -- hydrogen and argon, the dependence of the argon ion temperature on the magnetic field strength is resonant in nature with a maximum close to the magnetic field strength corresponding to the cyclotron value for protons. The maximum temperature of argon ions in the experiments was $2.5 \cdot 10^6$ °K, and for electrons -- $5 \cdot 10^5$ °K. In order to determine the distribution of the plasma ion temperature over the radius of the system, different sections of the plasma were focused on the spectrograph slit. The hottest plasma was located in the center of the filament. The temperature rapidly decreased /37 over the radius (approximately 5 times greater along the axis than on a radius of about 4 cm). Figure 8 illustrates the temperature dependence of

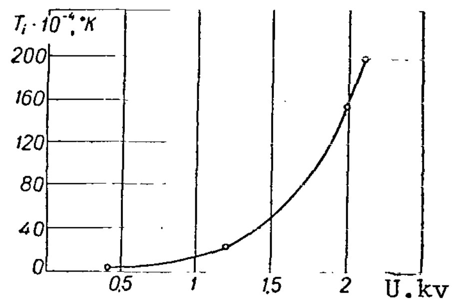

Figure 8

plasma on the applied high frequency voltage. The increase in the magnetic field strength and the high frequency voltage -- i.e., the hf power introduced -- made it possible to obtain a higher plasma temperature.

The mechanism by which the proton energy is transmitted to the additional gas has still not been definitely clarified. Since the gasokinetic pressure in these experiments may exceed the magnetic pressure, the possibility is not excluded that centrifugal instability may be produced, which can lead to transmission of proton energy to the additional gas and can lead to its heating.

In certain operational regimes of the apparatus, we observed generation of a rapid magnetosound wave at a magnetic field strength which was less than the cyclotron value for protons. However, conditions were not favorable for studying it at the existing oscillator frequency ($1.82 \cdot 10^6$ cps).

Thus, in all probability, these experiments illustrate the feasibility of high frequency heating of a dense plasma consisting of two types of ions by resonance generation of ion cyclotron waves for one type of ions. The mechanism by which energy is transmitted from one type of ions (protons) to other ions (helium, argon, admixture) requires further study.

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/38

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HIGH FREQUENCY PLASMA HEATING

K. N. Stepanov

As is well known, a rapid decrease in the frequency of Coulomb collisions of plasma particles leads to a decrease in the rate of ohmic plasma heating both by constant fields and by high frequency electric fields when there is a temperature increase. Therefore, great hopes have been expressed (Ref. 1, 2) for achieving plasma thermonuclear temperatures by utilizing different effects of collisionless energy absorption of high frequency fields by a plasma. In principle, this amounts to Cherenkov, or to cyclotron, absorption and radiation of waves by electrons and ions.

However, collisionless energy absorption by a plasma leads to great distortion of the velocity distribution function of electrons and ions and to attenuation, or even complete discontinuance, of absorption. It also leads to every type of plasma instability. If collisionless plasma heating is effectuated by weak electric fields, so that Coulomb collisions provide a Maxwell distribution function, the heating rate is the same as for ohmic heating. The time required to heat the plasma up to thermonuclear temperatures with weak fields is less than the time required for containing the plasma in a thermonuclear reactor, utilizing the reaction D + D, with a positive energy output. Therefore, under the condition of producing a stable, plasma configuration, it is primarily possible to achieve thermonuclear plasma temperatures with slow heating by weak, constant fields or by high frequency fields, which in themselves do not lead to strong plasma /39
instability.

On the other hand, for rapid plasma heating it is very tempting to employ strong electric fields, under whose influence electrons or ions in the plasma acquire a large directed velocity (Ref. 3 - 6), as well as strong electron bundles (Ref. 7) or the collision of plasma clusters (Ref. 3) ("turbulent" methods of plasma heating). Due to the development of bunched instability in the systems, there is a rapid increase in the energy of high frequency oscillations leading to braking of the accelerated particles, to an energy increase with respect to motion ("temperature"), and to an energy exchange between different plasma components.

This article presents a brief survey of the collisionless, high frequency methods of plasma heating, and it also compares them with the ohmic heating method.

Ohmic Plasma Heating

When investigating the processes of plasma heating, we shall disregard energy losses due to cyclotron radiation of electrons, which is absorbed by a plasma if its dimensions are fairly large, as well as losses due to plasma thermal conductivity on the chamber walls -- which is also a surface effect. In addition, we shall assume that the residual gas pressure is small, so that we can disregard energy losses due to overloading and radiation of admixed atoms excited by electron collision. Under these conditions, the energy losses by the plasma are determined only by braking radiation of the electrons.

When the ohmic method is employed to heat a plasma by a "constant" electric field with the strength E, the electron energy increase is determined by the following equation

$$\frac{dw}{dt} = Q_+ - Q_-, \tag{1}$$

where $w = \frac{3}{2} T_e$ is the mean electron energy; T_e -- electron temperatures;

$$n_0 Q_+ = \sigma E^2 \tag{2}$$

-- Joule heat liberated per unit volume per unit of time; n_0 -- electron density; $\sigma = \frac{1.96 e^2 n_0 \tau_e}{m_e}$ -- plasma conductivity; $\tau_e = \frac{3\sqrt{m_e} T_e^{3/2}}{4\sqrt{2\pi} n_0 e^4 \Lambda}$ -- time of /40

electron mean free path; Λ -- Coulomb logarithm. The intensity of electron braking radiation (Ref. 8) is

$$Q_- = \frac{32\sqrt{2T_e} e^6}{3\sqrt{\pi m_e} \hbar c^3 m_e}. \tag{3}$$

If we take into account the transmission of energy to ions, then we must replace Q_+ by $\frac{1}{2} Q_+$ in the case of $T_e \approx T_i$. This effect, as well as other effects which do not change in order of magnitude, is not taken into account from this point on.

Under the influence of the electric field E, the electrons acquire a directed velocity

$$u=\frac{\sigma E}{en_0}.$$

If the field strength E exceeds the critical value ($E_{cr} \sim \frac{en_0 v_e}{\sigma}$, where $v_e=\sqrt{\frac{T_e}{m_e}}$ is the electron thermal velocity), then all of the electrons are carried along by the electric field in a continuous acceleration regime. The motion of the electron gas with respect to the ions leads to the phenomenon of bunched instability which is related to the buildup of longitundinal high frequency oscillations in the plasma (Ref. 9). The inverse influence of plasma oscillations on the electron motion leads to electron braking -- i.e., to anomalous plasma resistance, and also to increased radiation of radio waves, which considerably increases the plasma thermal radiation (Ref. 10, 11).

Plasma bunched instability can arise in the case of $E \ll E_{cr}$, i.e., in the case of $u \ll v_e$. For example, in a very non-isothermic plasma, in the case of $T_e \gg T_i$ (T_i -- ion temperature) the electrons build up sound oscillations (Ref. 12), if the velocity u exceeds the speed of sound $V_s=\sqrt{\frac{T_e}{m_i}}$. In the case of $T_e \lesssim T_i$, the electrons build up longitudinal ion cyclotron oscillations, if $u > 10_{v_i}$ ($v_i = \sqrt{\frac{T_i}{m_i}}$ -- thermal ion velocity) (Ref. 13). However, it may be expected that -- since only a small group of resonance electrons participates in the oscillation buildup -- the formation of a "plateau" in the electron distribution function will lead to a decrease in the increasing increment (Ref. 14 - 18), and nonlinear effects will lead to stabilization of these oscillations (whose amplitude will be small) (Ref. 16 - 18). /41

Expression (2) may be employed only in the case of $E \ll E_{cr}$. Assuming that $E = \alpha E_{cr}$, where $\alpha \ll 1$, we obtain

$$Q_+ \sim \frac{\alpha^2 T_e}{\tau_e}. \qquad (4)$$

Let us substitute expressions (3) and (4) in (1). We then have

$$\frac{dw}{dt}=\left(\frac{3\cdot 10^{-9}\alpha^2}{\sqrt{T_e}}-5\cdot 10^{-15}\sqrt{T_e}\right)n_0 \text{ kev/sec}. \qquad (5)$$

where T_e -- in kev, n_0 -- in cm^{-3}. The heading process terminates, $\frac{dw}{dt} = 0$ in the case of

$$T_{max} \sim 5 \cdot 10^5 \alpha^2 \text{ kev} \tag{6}$$

It thus follows that even in the case of heating with weak fields ($\alpha = 10^{-1} - 10^{-2}$) it is possible to achieve thermonuclear temperatures ($T \sim 50$ kev).

If $Q_- \ll Q_+$, then -- disregarding Q_- as compared with εQ_+ in (1) and taking into account (4) -- we obtain

$$T_e = T_0 \left(1 + \frac{\alpha^2 t}{\tau_0}\right)^{2/3}, \tag{7}$$

where T_0 and τ_0 -- are the initial values of T_e and τ_e. The influence of braking radiation on the heating process is significant in the case of $T_e \sim T_{max}$, i.e., in the case of

$$t \sim \tau_0 \alpha \left(\frac{5 \cdot 10^5}{T_0}\right)^{3/2}. \tag{8}$$

For example, in the case of $T_0 \sim 100$ ev, $n_0 \sim 10^{15}$ cm^{-3}, $\tau_0 \sim 2 \cdot 10^{-8}$ sec and $\alpha \sim (1/30)$, the temperature $T_e = 50$ kev is achieved during the time $t \sim 0.2$ sec

$$t \sim \frac{\tau_0}{\alpha^2} \left(\frac{T_e}{T_0}\right)^{3/2}, \tag{9}$$

and the electric field strength changes between $0.3 - 3 \cdot 10^{-4}$ v/cm. The energy exchange between ions and electrons takes place during the time $\tau_{ie} \sim 0.5$ sec

$$\tau_{ie} \sim \tau_e \frac{m_i}{m_e}, \tag{10}$$

so that the separation between the electron temperature and the ion temperature is small. /42

The heating time (9) is not large from the point of view of producing thermonuclear reactors with a positive energy balance. The time the plasma is contained in such a reactor, when employing the reaction D + D, must be greater than (Ref. 19)

$$t^* \sim \frac{10^{16}}{n_0} \text{ sec.} \tag{11}$$

For the example under consideration ($n_0 \sim 10^{15}$ cm^{-3}) $t^* \sim 10$ sec, which exceeds the heating time of the ion component by a factor of 20.

Cherenkov Ion Heating

In the case of high frequency plasma heating employing the method advanced in (Ref. 20), oscillations of an axial magnetic field H_z which is produced by azimuthal electric currents, lead to a variable azimuthal electric field E_ϕ in a plasma cylinder located in a strong longitudinal field H_0. Radial, drift oscillations of the plasma arise, due to particle drift in crossed fields E_ϕ and H_0, which leads to density oscillations -- i.e., to the appearance of sound (more precisely, magnetosound) waves. (It is assumed that the wave frequency ω is considerably less than the ion gyrofrequency $\omega_i = \frac{eH_0}{m_i c}$, and the wave length is considerably less than the Larmor radius of ions $\rho_i = \frac{v_i}{w_i}$ with thermal velocity).

Ions having the velocity $v_{\parallel}$ along H_0, which is close to the phase velocity of a wave $V_\phi + \frac{\omega}{k_{\parallel}}$, vigorously interact with the field E_ϕ. If the wave phase velocity is on the order of the ion thermal velocity, the number of resonance ions is large, and there is strong wave absorption. (As is known, in the absence of a magnetic field sound oscillations in a plasma, in the case of $T_e \leq T_i$, cannot be propagated in general, due to strong Cherenkov absorption by ions (Ref. 21); in the case of $H_0 \neq 0$ and $T_e \leq T_i$, magnetosound oscillations are also damped during one period, if $V_\phi \sim v_i$ [Ref. 22]).

Let us investigate Cherenkov absorption by plasma ions of the energy of an electromagnetic field produced by azimuthal electric currents, /43
which take the form of a moving wave and flow into the coil placed on a plasma cylinder having the radius a:

$$j_\varphi = j_0 \cos(k_\parallel z - \omega t)\,\delta(r - a). \quad (12)$$

If $k_\parallel a \leq 1$, then the current (12) produces a variable longitudinal magnetic field with the strength

$$H_z = \tilde{H} \cos(k_\parallel z - \omega t), \quad (12')$$

where $\tilde{H} = \frac{4\pi j_0}{c}$. The strength of the aximuthal electric field is

$$E_\varphi = -\frac{\tilde{H}}{2n_\parallel} k_\parallel r \sin(k_\parallel z - \omega t),$$

where $n_\parallel = \frac{k_\parallel c}{\omega}$ is the longitudinal refractive index.

In the absence of damping, the value of the energy flux, averaged over time, in the plasma per unit length

$$S_0 = -\frac{c}{4\pi} E_\varphi H_z 2\pi a$$

equals zero. It is apparent that, in the presence of damping, the energy flux in the plasma equals, in order of magnitude,

$$S \sim \frac{ac}{4n_\parallel} \tilde{H}^2 \frac{\varkappa}{|n_\perp|}, \tag{13}$$

where $\varkappa$ is the correction to the "transverse" refractive index $n_\perp$, caused by the Cherenkov oscillation absorption by plasma ions.

The dispersion equation for a magnetosound wave has the form (Ref. 22)

$$n_\perp^2 + n_\parallel^2 = \frac{c^2}{V_A^2}\left(1 + i n_\perp^2 \frac{v_i^2}{c^2}\eta\right), \tag{14}$$

where the coefficient $\eta \sim 1$, if $V_\phi \sim v_i$. The component $\sim i$ takes into account Cherenkov wave damping in an ion gas.

Let us investigate a plasma with a small gasokinetic pressure $\left(\frac{8\pi n_0 T_{i,e}}{H_0^2} \ll 1\right)$. In this case, the Alfvén velocity $V_A = \frac{H_0}{\sqrt{4\pi n_0 m_i}}$ is considerably greater than the ion thermal velocity v_i and the speed of sound V_s. Since $n_\parallel \sim \frac{c}{v_i}$, in the zero approximation it follows from equation (14) that /44
$n_\perp = i n_\parallel$. In the following approximation we obtain: $n_\perp = i n_\parallel \left(1 + \frac{1}{2} i n_\parallel^2 \frac{v_i^2}{c^2}\eta\right)$, i.e.

$$\frac{\varkappa}{|n_\perp|} \sim \frac{8\pi n_0 T_i}{H_0^2}. \tag{15}$$

Taking into account expressions (13) and (15), we obtain the following formula for the mean increase in plasma ion energy $\frac{dw}{dt} = \frac{S}{\pi a^2 n_0}$ (Ref. 23)

$$\frac{dw}{dt} \sim \left(\frac{\tilde{H}}{H_0}\right)^2 \omega T_i. \tag{16}$$

In actuality, the energy is absorbed by resonance ions with $v_\parallel \approx \approx v_{res} = \frac{\omega}{k_\parallel}$. These ions may be regarded as magnetic dipoles with the magnetic moment $\mu = \frac{m_i v_\perp^2}{2H_0} \sim \frac{T_i}{H_0}$ which moves in the wave magnetic field with a strength which slowly changes in time and space. Within the frame of reference in which the wave is at rest, the equation of dipole motion

$$m_i \ddot{z} = -\mu \frac{\partial H_z}{\partial z} \tag{17}$$

leads to the law of energy conservation

$$\frac{m_i \dot{z}^2}{2} + \mu H_z = \text{const.} \tag{18}$$

Ions with velocities $v_\parallel$ in the $\frac{\omega}{k_\parallel} - \Delta v < v_\parallel < \frac{\omega}{k_\parallel} + \Delta v$ range, where

$$\Delta v = \sqrt{\frac{2\mu H_z}{m_i}}, \tag{19}$$

are trapped in the potential well and effectively interact with the wave. It is apparent that the number of resonance particles per unit volume is

$$n_{res} \sim n_0 \frac{\Delta v}{v_i} \sim n_0 \sqrt{\frac{\tilde{H}}{H_0}}. \tag{20}$$

Due to the shift of ions under the influence of the force $-\mu \frac{dH_z}{dz}$, the distribution function in the resonance region is distorted during the time $\tau_{nonlin.}$, during which a trapped particle covers a distance on the order of the potential well width, i.e., /45

$$\tau_{rel} \sim \frac{1}{k_\parallel \Delta v}. \tag{21}$$

Relaxation of the distribution function due to collisions in a narrow region of the width $\sim \Delta v$ close to $v_\parallel = \frac{\omega}{k_\parallel} \sim v_i$ occurs during the time period

$$\tau_{rel} \sim \tau_i \left(\frac{\Delta v}{v_i}\right)^2, \tag{22}$$

where

$$\tau_i = \frac{3\sqrt{m_i}\, T_i^{3/2}}{4\sqrt{2\pi}\, n_0 e^4 \Lambda}.$$

It is apparent that the ion distribution function will be distorted by an insignificant amount if $\tau_{nonlin.} \gg \tau_{rel}$. The critical value of the variable magnetic field strength $H = H_{cr}$, at which the distribution function distortion becomes significant, is determined from the condition $\tau_{nonlin.} \sim \tau_{rel}$. Taking into account (19), (21), and (22), we obtain (Ref. 23)

$$H_{cr} \sim \frac{H_0}{(\omega \tau_i)^{2/3}}. \tag{23}$$

Expression (23) for H_{cr} can be obtained by another method. An increase in the perpendicular velocity component (or a change in the magnetic moment) in the presence of a variable magnetic field is determined in the drift approximation from the following equation

$$\dot{v}_\perp = \frac{v_\perp \omega}{2H_0 k_\parallel} \cdot \frac{\partial H_z}{\partial z}. \tag{24}$$

The time of nonlinear distortion of the distribution function with respect to the velocity $v_\perp$ under the influence of the field E_ϕ -- i.e., under the influence of the force $\sim \frac{\partial H_z}{\partial z}$ in (24) -- is determined according to the relationship

$$\tau_{nonlin} \sim \frac{1}{k_\parallel \Delta v_\perp}, \tag{25}$$

where

$$\Delta v_\perp \sim \sqrt{\frac{\tilde{H}}{H_0} \cdot \frac{\omega}{k_\parallel} v_i}, \quad v_\perp \sim v_i. \tag{26}$$

If the ions obtain a velocity increase $\sim \Delta v_\perp$, then the relaxation time of the velocity distribution function is determined by /46

$$\tau_{rel} \sim \tau_i \left(\frac{\Delta v_\perp}{v_i}\right)^2. \tag{27}$$

The critical value of the magnetic field strength $\tilde{H} = H_{cr}$, at which great distortion of the ion distribution function $f(v_\parallel, v_\perp)$ occurs close to $v_\parallel = \frac{\omega}{k_\parallel}$, which is determined from the condition $\tau_{nonlin.} \sim \tau_{rel}$, coincides with the value of (23) in the case of $\frac{\omega}{k_\parallel} \sim v_i$. It is also apparent that the distortion of the velocity distribution function of the particles close to $v_\parallel = \frac{\omega}{k_\parallel}$ is not significant, if the energy acquired by resonance particles having the velocity $v_\parallel$ (in the $\frac{\omega}{k_\parallel} - \Delta v_\perp < v_\parallel < \frac{\omega}{k_\parallel} + \Delta v_\perp$ interval), during the time between two collisions τ_i

$$\left.\frac{dw}{dt}\right|_{res} \tau_i \sim \frac{dw}{dt} \cdot \frac{v_i}{\Delta v_\perp} \tau_i \sim \left(\frac{\tilde{H}}{H_0}\right)^{3/2} \omega T_i \tau_i, \tag{28}$$

is small as compared with their thermal energy T_i. The critical field strength $\tilde{H}$, determined from condition $\left.\frac{dw}{dt}\right|_{res} \tau_i \sim T_i$, coincides with (23).

During heating with fields $\tilde{H} = \alpha H_{cr}$ under the condition $\frac{\omega}{k_\parallel} \sim v_i$ (it is apparent that the fulfillment of this condition requires that the frequency be increased proportionally to $\sqrt{T_i}$ as the temperature increases)

$$\frac{dw}{dt} \sim \frac{\alpha^2 T_i}{\tau_i (\omega\tau_i)^{1/3}} = \frac{\alpha^2 T_0}{\tau_0 (\omega_0\tau_0)^{1/3}} \left(\frac{T_0}{T_i}\right)^{1/6}, \tag{29}$$

where T_0, τ_0 and ω_0 are the initial values of T_e, τ_e and ω.

Assuming that $w \sim T_i$ and integrating equation (29), we obtain the following expression for the ion temperature (Ref. 23)

$$T_i = T_0 \left[1 + \frac{\alpha^2 t}{\tau_0 (\omega_0\tau_0)^{1/3}}\right]^{6/11} \approx T_0 \left[1 + \frac{\alpha^2 t}{\tau_0 (\omega_0\tau_0)^{1/3}}\right]^{1/2}. \tag{30}$$

Plasma heating by means of ion Cherenkov resonance may be intensified, if -- instead of one wave -- several waves are employed with phase velocities differing by 2 - 3 Δv, so that all particles in the $-v_i \lesssim v_\parallel \lesssim v_i$ range may be resonance particles. In order to do this, it is sufficient to produce a wide wave packet with a phase velocity scattering of $\Delta(\frac{\omega}{k_\parallel}) \sim v_t$. It is apparent that the total number of such waves is $\frac{v_i}{\Delta v} \sim \sqrt{\frac{H_0}{H}}$. It is also evident that in this case the critical strength of the variable magnetic field responsible for great distortion of the ion distribution function over the entire $|\vec{v}| \lesssim v_i$ range is /47

$$H_{cr.tot.} \sim \frac{v_i}{\Delta v} H_{cr} \sim \sqrt{H_{cr} H_0} \sim \frac{H_0}{(\omega\tau_i)^{1/6}}. \tag{31}$$

In the case of heating by the field $H_z \sim \alpha H_{cr.\ tot.}$ ($\tilde{H} \sim \alpha^2 H_{cr}$) in the case of a wide wave packet, we have

$$\frac{dw}{dt} \sim \frac{v_i}{\Delta v} \left(\frac{\tilde{H}}{H_0}\right)^2 \omega T_i \sim \frac{\alpha^3 T_i}{\tau_i}. \tag{32}$$

Integrating expression (22), we obtain

$$T_i = T_0 \left(1 + \frac{\alpha^3 t}{\tau_0}\right)^{2/3}. \tag{33}$$

Formulas (32) and (33), which determine the ion temperature during the heating of plasma ions under optimum conditions, when practically all the plasma ions participate in absorption of the high frequency field energy, are similar to formulas (1), (4), and (7), which determine Joule electron heating by subcritical "constant" fields.

In the case of $\alpha = 0.01$, $n_0 \sim 10^{15}$ $1/cm^3$, $T_0 = 100$ ev, $H_0 \sim 10^5$G, and $\omega_0 \sim 10^7$ sec^{-1}, we obtain: $\tau_0 \sim 10^{-6}$ sec, $T_i = 50$ kev during the time $t \sim 1$ sec, while in the case of $t = 0$ $H_z \sim 500$G, $E_\phi \sim 50$ v/cm at the end of heating $\omega \sim 20\ \omega_0 \sim 2 \cdot 10^8$ $sec^{-1} \sim 0.2\omega_i$ and $H_z \sim 20$G, $E_\phi \sim 40$ v/cm ($E_\phi \approx \left(\frac{T_0}{T_i}\right)^{1/6} E_\phi \Big|_{t=0}$ barely changes). One advantage of this method is the fact that energy is transmitted directly to the plasma ion component.

In the case of a very non-isothermal plasma ($T_e >> T_i$), as was shown in (Ref. 24), a resonance relationship between the outer circuit and the plasma can exist in the case of $V_\phi \approx V_s$. In this case for $T_e \lesssim 10T_i$ ion absorption is still significant (on the order of electron absorption), and resonance $V_\phi \approx V_s$ can exist. The energy absorbed at the maximum increases by 10 - 100 times as compared with (16) (Ref. 23). /48

An expression for the high frequency power absorbed by a nonuniform plasma cylinder in the case of $\frac{\omega}{k_{||}} \sim v_i$ was obtained in (Ref. 25).

As of the present, the effect of Cherenkov absorption of a magnetosound wave by plasma ions has not been studied experimentally.

Cherenkov Electron Heating By the Field of a Magnetosound Wave (Helicons)

A rapid magnetosound wave can be propagated in a plasma with a large density ($\Omega_e^2 \gg \omega\omega_e$, where $\Omega_e = \sqrt{\frac{4\pi e^2 n_0}{m_e}}$ -- Langmuir frequency, $\omega_e = \frac{eH_0}{m_e c}$ -- electron gyrofrequency); the refractive index of this wave in the $\omega_i << \omega << \omega_e$ frequency region is determined by the expression

$$n \sim \frac{\Omega_e}{\sqrt{\omega\omega_e}} \sim \frac{c}{V_A}\sqrt{\frac{\omega_i}{\omega}} \tag{34}$$

(in this frequency region, rapid magnetosound waves are called "whistling atmospherics", or simply "atmospherics", "whistles", and "spiral waves").

Cherenkov whistle absorption by plasma electrons is weak not only for $\frac{\omega}{k_{||}} >> v_e$, when the damping coefficient is exponentially small (Ref. 26, 27), but also for $\frac{\omega}{k_{\|}} \lesssim v_e$. In the last case (Ref. 28, 29), we have

$$\frac{\varkappa}{n_{\perp}} \sim \frac{k_{\|} v_e}{\omega_e}. \tag{35}$$

Since the whistle field readily penetrates a dense plasma, these whistles may be employed to heat the plasma electron component, while electrons located within the plasma cylinder will be heated up due to the comparatively small absorption coefficient (35). /49

If $k_{\|} a \sim 1$, then -- substituting (35) in equation (14) -- we obtain

$$\frac{dw}{dt} \sim \left(\frac{\tilde{H}}{H_0}\right)^2 \omega T_e \left(\frac{\omega_e}{\omega} \cdot \frac{8\pi n_0 T_e}{H_0^2}\right)^{1/2}. \tag{36}$$

Since the factor $\left(\frac{\omega_e}{\omega} \cdot \frac{8\pi n_0 T_e}{H_0^2}\right)^{1/2}$ differs very little from unity in order of magnitude, this formula coincides with formula (16) (substituting T_e by T_i). However, since $\omega >> \omega_i$ in the case under consideration, and since formula (16) was obtained for the frequencies $\omega << \omega_i$, for the same values of $\tilde{H}$ electron heating by the whistle field takes place much more rapidly than Cherenkov heating of ions by a low frequency field.

The change in the electron distribution function with respect to $v_{\perp}$ by the quantity

$$\Delta v_{\perp} \sim v_e \sqrt{\frac{\tilde{H}}{H_0} \cdot \frac{\omega}{k_{\|} v_e}} \tag{37}$$

is insignificant, if the time of nonlinear distortion of the distribution function $\tau_{nonlin.} \sim \frac{1}{k_{\|} \Delta v_{\perp}}$ is a little greater than the relaxation time $\tau_{rel} \sim \tau_e \left(\frac{\Delta v_{\perp}}{v_e}\right)^2$. The critical strength of a variable magnetic field $\tilde{H} = H_{cr}$, at which the collisions cannot equalize the distribution function, is determined from the condition $\tau_{nonlin.} \sim \tau_{rel}$:

$$H_{cr} \sim \frac{H_0}{(\omega\tau_e)^{2/3}} \left(\frac{k_{\|} v_e}{\omega_e}\right)^{1/3} \sim \frac{H_0}{(\omega\tau_e)^{2/3}} \left(\frac{\omega_e}{\omega} \cdot \frac{4\pi n_0 T_e}{H_0^2}\right)^{1/6}. \tag{38}$$

This expression for H_{cr} is obtained under the condition that resonance electrons $\left(\frac{\omega}{k_{\|}} - \Delta v_{\perp} < v_{\|} < \frac{\omega}{k_{\|}} + \Delta v_{\perp}\right)$, which acquire the following energy per unit of time

$$\left.\frac{dw}{dt}\right|_{\text{res}} \sim \frac{dw}{dt}\cdot\frac{v_e}{\Delta v_\perp} \sim \left(\frac{\tilde{H}}{H_0}\right)^{3/2} \omega T_e \left(\frac{\omega_e}{\omega}\cdot\frac{8\pi n_0 T_e}{H_0^2}\right)^{1/4}, \tag{39}$$

If $\tilde{H} = H_{cr}$ during the time $\sim \tau_e$, acquire energy which is on the order of the thermal energy. /50

During electron heating by whistle fields with $\tilde{H} = \alpha H_{cr}$, the energy acquired by one electron on the average per unit of time is

$$\frac{dw}{dt} \sim \frac{\alpha^2 T_e}{\tau_e (\omega\tau_e)^{1/3}} \left(\frac{\omega}{\omega_e}\cdot\frac{H_0^2}{8\pi n_0 T_e}\right)^{1/4}. \tag{40}$$

Assuming, for purposes of simplicity, that $\left(\frac{\omega}{\omega_e}\cdot\frac{H_0^2}{8\pi n_0 T_e}\right)^{1/4} \sim 1$, we obtain

$$T_e = T_0 \left[1 + \frac{\alpha^2 t}{\tau_0 (\omega\tau_0)^{1/3}}\right]^{3/2}. \tag{41}$$

Under optimum conditions, when a wide wave packet is employed and when a large portion of electrons with a velocity of $|v_\parallel| \lesssim v_e$ are resonance electrons, electron heating by a field with $H_2 \sim \alpha H_{cr.\ tot.} \sim \alpha\sqrt{\tilde{H}_{cr} H_0}$ is determined by the following expression

$$\frac{dw}{dt} \sim \frac{\alpha^3 T_e}{\tau_e}; \quad T_e = T_0 \left(1 + \frac{\alpha^3 t}{\tau_0}\right)^{2/3}. \tag{42}$$

These formulas describe Cherenkov electron heating by a whistle field under optimum conditions, if all the plasma electrons participate in energy absorption. They are similar to formulas (4) and (7), which may be used to determine the electron temperature increase during ohmic heating by fields with subcritical strength.

The study (Ref. 30) was devoted to a theoretical investigation of Cherenkov electron heating by the electric field of a rapid magnetosound wave propagated in a nonuniform plasma cylinder. Cherenkov whistle absorption was determined experimentally in (Ref. 31).

Ion Cyclotron Resonance. Absorption of Alfvén Wave

Ions are heated most effectively by the field of a high frequency wave under conditions of ion cyclotron resonance. If the frequency of a wave propagated in a low-pressure plasma is close to the ion cyclotron frequency, then there will be a large number of particles having the velocity $v_{\parallel}$, which is close to the particle resonance velocity /51

$$v_{\text{res}} = \frac{\omega - \omega_i}{k_{\parallel}}, \tag{43}$$

and effectively interacting with the wave field. Therefore, the wave energy absorption by ions will be large. Cyclotron damping of waves is the inverse of cyclotron radiation of waves by charged particles in a magnetic field. This effect was first studied in (Ref. 32), where they investigated the damping of magnetohydrodynamic waves propagated along the magnetic field. A study of the cyclotron absorption of waves having a frequency of $\omega \approx \omega_i$ was pursued in (Ref. 33) [see also the studies (Ref. 20, 34, 35)].

As is well known, in the frequency region on the order of ω_i in a cold plasma ($T_\alpha = 0$) there are two branches of oscillations corresponding to an Alfvén wave (which can only be propagated in the case of $\omega < \omega_i$) and corresponding to a rapid magnetosound wave. As the Alfvén wave frequency approaches ω_i, its refractive index and cyclotron damping coefficient increase. In the case of $\omega_i - \omega \lesssim k_{\parallel} v_i$, the propagation of this wave by strong damping is impossible: $\mathrm{Re}\, k \sim \mathrm{Im}\, k \sim 1/\delta_i$, where δ_i is the depth to which the field penetrates the plasma (Ref. 28, 36, 37)

$$\delta_i = \frac{\left(v_i V_A^2\right)^{1/3}}{\omega_i}. \tag{44}$$

A magnetosound wave is absorbed slightly in the case of $|\omega - \omega_i| \lesssim k_{\parallel} v_i$.

Let us first examine cyclotron heating of a plasma cylinder by an Alfvén wave which is strongly damped. The azimuthal currents (12) excite this wave if $k_{\parallel} \delta_i \sim 1$. Also assuming that $k_{\parallel} a \sim 1$ and $\omega_i - \omega \lesssim k_{\parallel} v_i$, we obtained the following expression from formula (13) for the mean energy acquired by one ion per unit of time

$$\frac{dw}{dt} \sim \frac{\tilde{H}^2 \omega}{8\pi n_0} \sim \left(\frac{\tilde{H}}{H_0}\right)^2 \omega T_i \frac{H_0^2}{8\pi n_0 T_i}. \tag{45}$$

It thus follows that the electromagnetic field energy which is accumulated in the plasma $\left(\frac{\tilde{H}^2}{8\pi}\right)$ is absorbed during a period of time on the order of $\frac{1}{\omega_i}$.

In addition, a comparison of (45) and (16) shows that, for the same /52
variable magnetic field strength, the energy absorbed by the plasma in the case of $\omega \approx \omega_i$ is considerably greater than in the case of Cherenkov

resonance. In the first place, this is due to the difference in the frequencies $\left(\frac{dw}{dt}\right) \sim \omega$ and, in the second place, it is due to a large factor $\frac{H_0^2}{8\pi n_0 T_i}$ in expression (32). On the other hand, for the same values of $\tilde{H}$ and $k_{\parallel}$ the electric field strength in the case of cyclotron resonance is $\frac{\omega_i}{\omega_{Cher.}}$ times greater than in the case of Cherenkov resonance ($\omega_{Cher.}$ -- the wave frequency under Cherenkov resonance conditions).

In order to determine the number of resonance ions responsible for energy absorption during cyclotron resonance, let us investigate the particle motion in the field of a flat cyclotron wave:

$$H_x = \tilde{H}\sin(k_{\parallel}z - \omega t),\ H_y = -\tilde{H}\cos(k_{\parallel}z - \omega t);$$
$$E_x = -\frac{\tilde{H}}{n_{\parallel}}\cos(k_{\parallel}z - \omega t),\ E_y = -\frac{\tilde{H}}{n_{\parallel}}\sin(k_{\parallel}z - \omega t), \tag{46}$$

where $n_{\parallel} = \frac{k_{\parallel}c}{\omega} = \frac{c}{V_A\sqrt{\omega_i/\omega - 1}}$ is the refractive index.

In the absence of a wave, the ions move along a spiral:

$$\vec{v} = \vec{v}_0 = \left(v_{\perp}^0\cos(\omega_i t + \varphi_0),\ -v_{\perp}^0\sin(\omega_i t + \varphi_0),\ v_{\parallel}\right).$$

When there is a weak field, the velocity perturbation is determined according to the equations of motion

$$\dot{v}_x = -\frac{e\tilde{H}}{m_i n_{\parallel}}\cos\left(k_{\parallel}z - \omega t\right) + \omega_i v_y;$$
$$\dot{v}_y = -\frac{e\tilde{H}}{m_i n_{\parallel}}\sin\left(k_{\parallel}z - \omega t\right) - \omega_i v_x;$$
$$\ddot{z} = \frac{e}{m_i c}\left(v_x^0 H_y - v_y^0 H_x\right) = -\frac{e v_{\perp}^0 \tilde{H}}{m_i c}\cos\Phi,$$

where $\Phi = k_{\parallel}z + (\omega_i - \omega)t + \varphi_0$.

Assuming that $u = v_x + iv_y$, from the first two equations we obtain /53

$$\dot{u} + i\omega_i u = -\frac{e\tilde{H}}{m_i n_{\parallel}}e^{i(k_{\parallel}z - \omega t)}.$$

Thus, assuming that $u = u_0 e^{-i\omega_i t}$, we obtain

$$\dot{u}_0 = -\frac{e\tilde{H}}{m_i n_\parallel} e^{i(\Phi - \varphi_0)}.$$

The third equation can be written as follows

$$\ddot{\Phi} + \frac{ev_\perp^0 \tilde{H} k_\parallel}{m_i c} \cos\Phi = 0. \tag{47}$$

Let us first investigate nonlinear distortion of the distribution function close to $v_\parallel = v_{res}$, caused by a variable magnetic field. The law of energy conservation follows from (47)

$$\frac{1}{2}\dot{\Phi}^2 + \frac{ev_\perp^0 \tilde{H} k_\parallel}{m_i c} \sin\Phi = \text{const}.$$

The time of nonlinear distortion of the distribution function close to $v_\parallel = v_{res}$ equals the potential well flight time of trapped particles having the velocity $\sim \Delta v_\parallel = \sqrt{\frac{2ev_\perp^0 \tilde{H}}{mk_\parallel c}}$:

$$\tau_{nonlin} \sim \frac{1}{k_\parallel \Delta v_\parallel}.$$

We obtain the following expression from the condition $\tau_{nonlin.} \sim \tau_{rel} \sim \sim \tau_i \left(\frac{\Delta v_\parallel}{v_i}\right)^2$ for the critical value of the magnetic field strength

$$H_{cr} \sim \frac{H_0}{(\omega\tau_i)^{2/3}} \left(\frac{8\pi n_0 T_i}{H_0^2}\right)^{1/3}. \tag{48}$$

This expression may also be obtained from the formula for the nonlinear decrement of cyclotron wave damping, which is determined on the basis of the quasilinear theory (Ref. 38) in the case of $v_{res} \gg v_i$, if we set $v_{res} \sim v_i$ and $k_\parallel \sim \frac{1}{\delta_i}$ in formula (25) of the study (Ref. 38).

However, the influence of the accelerating field $\vec{E}$ on the distribution function change in a plane perpendicular to H_0 is considerably stronger than the influence of a variable magnetic field on the distribution function change with respect to velocity along H_0. /54

Let us determine the time of nonlinear distortion of the distribution function by the field $\vec{E}$:

$$\tau_{nonlin} \sim \frac{1}{k_\parallel \Delta v_\perp},$$

where

$$\Delta v_{\perp} \sim v_i \sqrt{\frac{\widetilde{H}}{H_0} \cdot \frac{\omega}{k_{\parallel} v_i}} \sim v_i \sqrt{\frac{\widetilde{H}}{H_0}} \left(\frac{H_0^2}{8\pi n_0 T_i} \right)^{1/3}. \tag{49}$$

Equating $\tau_{\text{nonlin.}}$ and $\tau_{\text{rel}} \sim \tau_i \left(\frac{\Delta v_{\perp}}{v_i} \right)^2$ in the case of $\widetilde{H} \sim H_{cr}$, we obtain

$$H_{cr} \sim \frac{H_0}{(\omega\tau_i)^{2/3}} \left(\frac{8\pi n_0 T_i}{H_0^2} \right)^{4/9}. \tag{50}$$

We obtain the same expression by assuming that only a group of resonance particles with $v_{res} - \Delta v_{\perp} < v_{\parallel} < v_{res} + \Delta v_{\perp}$ participates in energy absorption. Resonance particles acquire the following energy per unit of time

$$\left. \frac{dw}{dt} \right|_{res} \sim \left(\frac{\widetilde{H}}{H_0} \right)^{3/2} \omega T_i \left(\frac{H_0^2}{8\pi n_0 T_i} \right)^{2/3}.$$

In the case of $H \sim H_{cr}$, during the time $\sim \tau_i$ these particles collect the energy $\sim T_i$. The critical field strength (50) is considerably less than (48). This means that the influence of nonlinear effects caused by the electric field is manifested for smaller field strengths than is the case for the influence of a variable magnetic field.

In the case of $\widetilde{H} = \alpha H_{cr}$, we obtain the following expression for $\frac{dw}{dt}$

$$\frac{dw}{dt} \sim \frac{\alpha^2 \omega T_i}{(\omega\tau_i)^{4/3}} \left(\frac{H_0^2}{8\pi n_0 T_i} \right)^{1/9} = \frac{\alpha^2 T_0}{\tau_0 (\omega\tau_0)^{1/3}} \left(\frac{H_0^2}{8\pi n_0 T_0} \right)^{1/9} \left(\frac{T_0}{T_i} \right)^{10/9}. \tag{51}$$

Thus, we have

$$T_i = T_0 \left[1 + \frac{\alpha^2 t}{\tau_0 (\omega\tau_0)^{1/3}} \left(\frac{H_0^2}{8\pi n_0 T_i} \right)^{1/9} \right]^{9/10} \approx T_0 \left[1 + \frac{\alpha^2 t}{\tau_0 (\omega\tau_0)^{1/3}} \right]^{1/2}. \tag{52}$$

Let us now present a numerical example. Let us set $n_0 \sim 10^{13}$ cm^{-3}, /55
$H_0 \sim 5 \cdot 10^3$G, $\omega_i \sim 5 \cdot 10^7$ sec^{-1} and $\alpha \sim 1$. Then in the case of $T_i = 10$ ev, we obtain: $\tau_i \sim 3 \cdot 10^{-6}$ sec, $\widetilde{H} \sim 2$G and $\frac{dw}{dt} \sim 300$ kev/sec; if $T_i \sim 100$ ev, then $\tau_i \sim 10^{-4}$ sec, $\widetilde{H} \sim 0.4$G and $\frac{dw}{dt} \sim 30$ kev/sec; if $T_i \sim 1$ kev, then $\tau_i \sim 3 \cdot 10^{-3}$ sec, $\widetilde{H} \sim 0.1$G and $\frac{dw}{dt} \sim 3$ kev/sec.

In the case of $T_0 \sim 100$ ev and $\widetilde{H} \sim H_{cr}$, the temperature $T_i \sim 10$ kev is achieved during the time $t \sim 3$ sec.

Cyclotron heating by fields with subcritical strengths may be intensified, if several waves (wave packet) are employed having phase velocities which differ with respect to 2 - 3 $\Delta v_{\perp}$. The number of such waves is $\sim \frac{v_i}{\Delta v_{\perp}}$, and the critical strength of the total magnetic field is $H_{cr.\ tot} \sim \sqrt{H_{cr} H_0}$, where H_{cr} is determined according to (50). In the case of heating by the field $H_z \sim \alpha H_{cr.\ tot}$, in this case for $\frac{dw}{dt}$ we obtain (32), and for T_i we obtain (33).

Cyclotron wave absorption in a plasma cylinder with a constant (over the cross section) plane and temperature was analyzed in (Ref. 33, 39). The absorption of long wave oscillations in a nonuniform plasma cylinder was studied in (Ref. 40).

In the case of a plasma with a large density and magnetic fields with a high strength, the skin depth δ_i (44) is small. When δ_i is less than the plasma radius, plasma heating by an Alfvén wave is ineffective, since the wave energy is absorbed only by ions located on the periphery of the plasma cylinder.

In order to avoid this difficulty, Stix (Ref. 41) employed the ingenious idea of "magnetic beaches". If the plasma cylinder is placed in the field H_0 with a slowly decreasing strength, then cyclotron damping is exponentially small in the region $\omega_i - \omega \gg k_{\parallel} v_i$. An Alfvén wave readily penetrates the plasma, and they may be excited resonantly. When the wave is propagated along H_0, the difference $\omega_i - \omega$ decreases, while the refractive index $n_{\parallel} \sim 1/\sqrt{\omega_i - \omega}$ and the damping coefficient $\varkappa \sim \exp\left[\frac{\omega_i^2 - \omega^2}{2k_{\parallel}^2 v_i^2}\right]$ /56 increase. If the magnetic field decreases smoothly, the reflection coefficient will be small, and the wave will be absorbed at the approach to the region of strong cyclotron damping, where $\omega_i - \omega \lesssim k_{\parallel} v_i$ (region of the "magnetic beach"). The field behavior close to the magnetic beach was determined in (Ref. 42, 43). Absorption of Alfvén waves at the magnetic beaches was determined experimentally in (Ref. 44). Experiments performed at Princeton closely coincide with the theoretical computations of cyclotron damping in a linear approximation [see the summary in (Ref. 45)]. On the other hand, in several devices for ion cyclotron heating (see, for example, [Ref. 46]) the variable field strengths are on the order of, or even considerably larger than, the critical strengths. Under these conditions, we must expect a decrease in the field energy absorption, as compared with the case of weak fields.

Cyclotron Absorption of a Magnetosound Wave

The above-mentioned difficulty entailed in high frequency heating of a plasma having a large density by a strongly damped Alfvén wave is unimportant for a rapid magnetosound wave. A magnetosound wave is absorbed in the case of $\omega \approx \omega_i$ to a considerably lesser extent, and can penetrate the plasma readily. The damping coefficinet of this wave in the case of $|\omega - \omega_i| \lesssim k_\| v_i$ is on the order of (Ref. 28)

$$\frac{\varkappa}{n_\perp} \sim \sqrt{\frac{8\pi n_0 T_i}{H_0^2}}, \tag{53}$$

where $n_\perp \sim \frac{c}{V_A}$.

If $k_{||} \alpha \sim 1$, we obtain the following expression (Ref. 39) for the energy absorbed on the average by one ion per unit of time from the formula (13), taking into account (53)

$$\frac{dw}{dt} \sim \left(\frac{\tilde{H}}{H_0}\right)^2 \omega T_i \left(\frac{H_0^2}{8\pi n_0 T_i}\right)^{1/2}. \tag{54}$$

Plasma heating by a magnetosound wave is greatly intensified, if the frequency $\omega \approx \omega_i$ coincides with the eigen oscillation frequency of a plasma cylinder ω_{res}. In this case, the field strength in the plasma increases /57

in the case of $|\omega_{res} - \omega| \lesssim \omega \sqrt{\frac{8\pi n_0 T_i}{H_0^2}}$, as compared with the non-resonance case, by a factor of $\sqrt{\frac{H_0^2}{8\pi n_0 T_i}}$:

$$H_z \sim \tilde{H} \sqrt{\frac{H_0^2}{8\pi n_0 T_i}}. \tag{55}$$

It is apparent that we then have

$$\frac{dw}{dt} \sim \left(\frac{\tilde{H}}{H_0}\right)^2 \omega T_i \left(\frac{H_0^2}{8\pi n_0 T_i}\right)^{3/2} \tag{56}$$

Multiple resonance: $\omega = 2\omega_i$, may be employed to heat a plasma with a large density by a magnetosound wave. The damping coefficient of the magnetosound wave in the case of $|\omega - 2\omega_i| \lesssim k_{||} v_i$ is (Ref. 28)

$$\frac{\varkappa}{n_\perp} \sim \sqrt{\frac{8\pi n_0 T_i}{H_0}} \left(\frac{\Omega_i}{k_{||} c}\right)^2, \tag{57}$$

where $\Omega_i = \sqrt{\frac{4\pi e^2 n_0}{m_i}}$ is the ion Langmuir frequency. It follows from (57) that in a plasma having a large density $\Omega_i > k_{\parallel} c$ resonance at the multiple frequency $\omega \approx 2\omega_i$ is more advantageous than at the main frequency.

The heating of a plasma cylinder which is uniform across its cross section was examined for $\omega \approx n\omega_i$ ($n = 2,3, \ldots$) in (Ref. 47), and the case of wavelengths ($k_{||} \alpha \ll 1$) in a nonuniform plasma was investigated in (Ref. 48). Cyclotron absorption of magnetosound waves in the case of $\omega = \omega_i$ and $\omega = 2\omega_i$ has not been determined experimentally as yet, although the inverse effect -- cyclotron radiation of ions in a dense plasma -- was studied recently (Ref. 49).

The statements presented above illustrate the following:

(1) Cherenkov and cyclotron plasma heating with weak fields, when distortion of the ion distribution function is compensated by collisions, occur at the same rate as ohmic heating by a "constant" electric field, /58
whose strength is less than the critical strength;

(2) Plasma heating by weak fields up to thermonuclear temperatures ($T_i \sim 50$ kev) takes place over a long period of time. However, this time is less than the time for containing a plasma in a thermonuclear reactor with a positive balance;

(3) When heating is performed with fields having subcritical strengths, it is more advantageous to employ a plasma with a great density, since collisions occur more frequently in it, the critical field strengths are larger, and the heating time is less than ($t \sim \frac{1}{n_0}$).

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DIELECTRIC CONSTANT OF A PLASMA IN A DIRECT PINCH MAGNETIC FIELD AND IN A DIRECT HELICAL MAGNETIC FIELD

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Dielectric Constant of a Plasma in a Direct Pinch Magnetic Field /60

The study of electromagnetic oscillations in a nonuniform plasma has aroused a great deal of interest recently. This is primarily due to the discovery of several instabilities. A great number of these studies* has been devoted to studying the electromagnetic properties of a slightly-nonuniform plasma located either in an almost uniform magnetic field with parallel force lines, or in an axially symmetrical magnetic field with helical magnetic force lines.

In conjunction with these cases, when investigating the problem of controlled thermonuclear synthesis it is very important to study the plasma electromagnetic oscillations in pinch magnetic fields and in helical magnetic fields and in a stellarator with helical current winding (Ref. 3), which has a more complex structure than magnetic force lines.

As is well known, the electromagnetic properties of media are described by the dielectric constant tensor or the electroconductivity tensor related to it. In slightly nonuniform media, one can introduce quantities which are similar to the electroconductivity tensor and the dielectric constant tensor. In contrast to a uniform plasma, these tensors depend on spatial variables in wave vector space $\mathbf{k}$ and the frequency ω.

If $\mathbf{E}(\mathbf{k},\omega)$ is the electric field strength, then the density of the current induced by this field can be written as follows

$$j_\alpha(\mathbf{r},\ t) = \int dk d\omega \sigma_{\alpha\beta}(\mathbf{k},\ \omega,\ \mathbf{r}) E_\beta(\mathbf{k},\ \omega) e^{i(\mathbf{kr}-\omega t)}, \tag{1}$$

where $\sigma_{\alpha\beta}(\mathbf{k},\ \omega,\ \mathbf{r})$ is the electroconductivity tensor of a nonuniform plasma, which is related to the dielectric constant tensor by the well-known relationship /61

$$\varepsilon_{\alpha\beta}(\mathbf{k},\ \omega,\ \mathbf{r}) = \delta_{\alpha\beta} + \frac{4\pi i}{\omega}\sigma_{\alpha\beta}(\mathbf{k},\ \omega,\ \mathbf{r}). \tag{2}$$

The explicit form of the tensor $\sigma_{\alpha\beta}(\mathbf{k},\ \omega,\ \mathbf{r})$ may be found by different

* A detailed list of the literature is presented in the review articles of A. B. Mikhaylovskiy (Ref. 1), A. A. Rukhadze, and V. P. Silin (Ref. 2).

methods. In our case, it is advantageous to employ the method advanced by V.D. Shafranov (Ref. 4). In order to do this, it is necessary to determine the trajectory of the unperturbed particle motion and to solve the kinetic equation by the characteristics method.

In the absence of a balanced electric field, the unperturbed charged particle motion may be described by the equations

$$\frac{d\mathbf{v}}{dt}=\frac{e}{mc}[\mathbf{vB}];\ \frac{d\mathbf{r}}{dt}=\mathbf{v}, \tag{3}$$

where $\mathbf{B}$ is the magnetic field strength in the plasma; e and m -- particle charge and mass, respectively. In the cylindrical coordinate system, the strength components of a nonvortical, direct pinch magnetic field have the following form

$$\begin{aligned} B_r &= B_0\sum_{n=1}^{\infty} g_n \sin \alpha z n; \\ B_z &= B_0\left(1+\sum_{n=1}^{\infty} f_n \cos \alpha n z\right); \\ B_\varphi &= 0, \end{aligned} \tag{4}$$

where B_0 is a uniform magnetic field; $\alpha = \frac{2\pi}{L}$; $f_n(r)$ and $g_n(r)$ -- functions of the coordinate r which are connected by the relationships

$$\frac{df_n}{dr}=n\alpha g_n;\ \frac{d(rg_n)}{dr}=rn\alpha f_n. \tag{5}$$

In the presence of a plasma, the field components, which are related to the pressure gradient and the longitudinal current and which may be found from the equations of plasma equilibrium, must be added to the nonvortical field (4).

A solution of equations (3) in the general case of arbitrary fields /62
entails considerable difficulties. Let us examine the case (which is of practical interest) of a low-pressure plasma $\left(\beta = \frac{8\pi p}{B^2} \ll 1\right)$ without a longitudinal current, in a pinch magnetic field (4) having a large uniform component $B_0 (f_n \sim g_n \sim \delta \ll 1)$.

In solving nonlinear equations (3), the presence of a small parameter δ makes it possible to employ -- along with the drift approximation -- the method of averaging (Ref. 5) when solving drift equations for flying particles. The number of particles which are blocked is small, and their contribution to the tensor $\sigma_{\alpha\beta}$ may be disregarded. Avoiding cumbersome computations, let us derive the final result of solving the equations of motion (3) within an accuracy of terms on the order of $\frac{\delta^2}{\omega_B}$ $\left(\omega_B = \frac{eB_0}{mc}\right)$ inclusively

$$r(t) = r - \sum_{n=1}^{\infty} \frac{g_n}{n\alpha} [\cos n\alpha (v_{\parallel} t + z) - \cos n\alpha z] - \frac{v_{\perp}}{\omega_B} [\sin (\gamma - \omega_B t) - \sin \gamma];$$

$$\varphi (t) = \varphi + \frac{\bar{v}_{\varphi} t}{r} + \sum_{n=1}^{\infty} \frac{2v_{\parallel}^2 + v_{\perp}^2}{2v_{\parallel} r\omega_B} g_n [\sin n\alpha (v_{\parallel} t + z) - \sin n\alpha z] + \frac{v_{\perp}}{r\omega_B} [\cos (\gamma - \omega_B t) - \cos \gamma];$$

$$z(t) = z + v_{\parallel} t - \sum_{n=1}^{\infty} \frac{v_{\perp}^2 f_n}{2n\alpha v_{\parallel}^2} [\sin n\alpha (v_{\parallel} t + z) - \sin n\alpha z]; \qquad (6)$$

$$v_{\parallel}(t) = v_{\parallel} - \sum_{n=1}^{\infty} \frac{v_{\perp}^2}{2v_{\parallel}} f_n [\cos n\alpha (v_{\parallel} t + z) - \cos n\alpha z];$$

$$v_{\perp}(t) = v_{\perp} + \sum_{n=1}^{\infty} \frac{1}{2} v_{\perp} f_n [\cos n\alpha (v_{\parallel} t + z) - \cos n\alpha z].$$

where $v_{\parallel}$ and $v_{\perp}$ are the velocity components which are longitudinal and /63
transverse to the field, respectively; γ -- the initial phase of particle rotation around the center of a Larmor circle; $\bar{v}_{\phi}$ -- the averaged velocity component related to the drift of the Larmor circle center:

$$\bar{v}_{\varphi} = \frac{v_{\perp}^2}{r\omega_B} u_1 + \frac{v_{\parallel}^2}{r\omega_B} u_2 + \frac{v_{\perp}^4}{r\omega_B v_{\parallel}^2} u_3;$$

$$u_1 = -\frac{r}{4} \sum_{n=1}^{\infty} g_n g_n'; \qquad u_3 = \frac{r}{8} \sum_{n=1}^{\infty} f_n f_n'; \quad u_2 = -\frac{r}{2} \sum_{n=1}^{\infty} g_n g_n' + \frac{4\pi p' r}{B_0^2}. \qquad (7)$$

For purposes of simplicity, we have omitted the index 0 for quantities taken at the initial moment of time t = 0 in formulas (6) and (7). We shall also do this from this point on.

By solving the kinetic equation, with no allowance for close collisions, by the method of characteristics, we can find the expression for the electroconductivity tensor in a cylindrical coordinate system from the equation for the induced current density

$$\sigma_{\alpha\beta}(\mathbf{k}, \omega, \mathbf{r}) = -\sum_{i,e} \frac{e^2}{m} \int d\mathbf{v} v_{\alpha} \int_{-\infty}^{0} \frac{\partial F_0}{\partial v_{\gamma}(t)} \left\{ \left(1 - \frac{\mathbf{k}\mathbf{v}(t)}{\omega}\right) \delta_{\gamma\beta} + \frac{k_{\gamma} v_{\beta}(t)}{\omega} - \frac{i}{r(t)\,\omega} [\mathbf{v}(t)\, \mathbf{e}_z]_{\gamma} \delta_{\beta\varphi} \right\} e^{-i\omega t + i\mathbf{k}[\mathbf{r}(t) - \mathbf{r}]} dt, \qquad (8)$$

where F_0 is the equilibrium distribution function of the particles; **k** -- the wave vector with the components k_r, k_z, k_ϕ ($k_\phi = \frac{m}{r}$, m -- whole numbers); $\sum_{i,e}$ -- the sum with respect to ions and electrons. The electric field may be determined by the expression

$$\mathbf{E}(\mathbf{r}, \omega) = \sum_m \int dk_r dk_z \, \mathbf{E}(k_r, k_z, m) \, e^{ik_r r + ik_z z + im\varphi}. \tag{9}$$

For a slightly nonuniform plasma in a strong magnetic field, we may write

$$F_0 = \left(1 + \frac{1}{\omega_B}[\mathbf{v}\vec{\tau}] \nabla \Psi \frac{\partial}{\partial \Psi}\right) f(v_\parallel, v_\perp, \Psi), \tag{10}$$ /64

for the equilibrium distribution function within an accuracy of terms on the first order of smallness with respect to the parameter $\frac{v_T}{\omega_B a}$ (v_T -- thermal particle velocity, a -- characteristic dimension of the nonuniformity for the main plasma state). Here, $\vec{\tau}$ -- unit vector in the field direction; f -- arbitrary function of the velocities $v_\parallel$, $v_\perp$ and of the variable Ψ which is an integral of the drift equations. Within an accuracy of the terms $\frac{1}{\omega_B}$, the quantity Ψ coincides with the integral of the force line equations

$$\Psi = r^2 + 2r \sum_{n=1}^{\infty} \frac{g_n}{n\alpha} \cos n\alpha z. \tag{11}$$

Due to the smallness of the parameter δ, we may approximately compute the function f which depends only on the coordinate r. We should note that, in order to avoid this, in a plasma with the selected distribution function $\operatorname{div} \mathbf{j} \neq 0$ we may add the component $\mathbf{vB} \times \int \frac{[\mathbf{B} \nabla B^2]}{\omega_B B^4} \nabla f dl$ to the function F_0, where integration is performed along the magnetic force line. However, as may be readily seen, in our approximation this term is small.

Let us select the Maxwell distribution of particles with a nonuniform equilibrium density n_0 and the temperature T, which depend on Ψ, as f:

$$f = n_0 \left(\frac{m}{2\pi T}\right)^{3/2} \exp\left(-\frac{m}{T} \cdot \frac{v_\parallel^2 + v_\perp^2}{2}\right). \tag{12}$$

Retaining the important terms in (8), after simple transformations we obtain

$$\sigma_{\alpha\beta}(k, \omega, r) = i\pi \sum_{i,e} e^2 \int \hat{M} \hat{L} \frac{1}{T} f(v_\parallel, v_\perp, \Psi) \, Q_{\alpha\beta} dv_\parallel dv_\perp^2, \tag{13}$$

where the following notation is employed:

$$\hat{M} = \prod_{n=1}^{\infty} \sum_{s,\, s'} J_s(\eta_n) J_{s'}(\eta_n) e^{i(s-s')\left(n\alpha z+\psi_n-\frac{\pi}{2}\right)};$$

$$\eta_n = \left\{\left(\frac{k_r g_n}{\alpha n}\right)^2 + \left[\frac{k_z v_\perp^2 f_n}{2n\alpha v_\parallel^2} - \frac{g_n k_\varphi \left(2v_\parallel^2 + v_\perp^2\right)}{2v_\parallel \omega_B}\right]^2\right\}^{1/2};$$

$$\psi_n = \operatorname{arctg} \frac{n\alpha\left[\frac{k_\varphi\left(2v_\parallel^2 + v_\perp^2\right) g_n}{2v_\parallel \omega_B} - \frac{k_z v_\perp^2 f_n}{2n\alpha v_\parallel^2}\right]}{k_r g_n},$$

$$\hat{L} = 1 - \frac{\mathbf{k}\,[\vec{\tau\nabla\Psi}]}{\omega\omega_B}\left(\frac{\partial T}{\partial \Psi}\cdot\frac{\partial}{\partial T} + \frac{\partial n_0}{\partial \Psi}\cdot\frac{\partial}{\partial n_0}\right) T; \qquad (14) \quad /65$$

$$Q_{\alpha\beta} = -\frac{i}{2\pi}\int_0^{2\pi} d\gamma v_\alpha \int_{-\infty}^{0} v_\beta(t)\, A(t)\, dt;$$

$$A(t) = \exp\{-i(\omega - n\alpha s v_\parallel - \bar{k}_\varphi v_\varphi - k_z v_\parallel)t - i\xi \times$$
$$\times [\sin(\gamma - \omega_B t - \psi) - \sin(\gamma - \psi)]\};$$

$$\psi = \operatorname{arctg}\frac{k_\varphi}{k_r}; \quad \xi = \frac{k_\perp v_\perp}{\omega_B}; \quad k_\perp = \sqrt{k_r^2 + k_\varphi^2}.$$

The velocity components which are included in the expression for the tensor $Q_{\alpha\beta}$ in a cylindrical coordinate system have the following form

$$v_r(t) = v_\perp \cos(\gamma - \omega_B t) + \sum_{n=1}^{\infty} g_n v_\parallel \sin n\alpha (v_\parallel t + z);$$

$$r\frac{d\varphi(t)}{dt} = v_\perp \sin(\gamma - \omega_B t) + \sum_{n=1}^{\infty} \frac{2v_\parallel^2 + v_\perp^2}{2\omega_B} f_n \cos n\alpha (v_\parallel t + z) + \bar{v}_\varphi; \qquad (15)$$

$$v_z(t) = v_\parallel - \sum_{n=1}^{\infty} \frac{v_\perp^2}{2v_\parallel} f_n \cos n\alpha (v_\parallel t + z).$$

By employing them, we can obtain the explicit form of the tensor $Q_{\alpha\beta}$

$$Q_{\alpha\beta} = \sum_{p=-\infty}^{\infty} q_\alpha \tilde{q}_\beta, \qquad (16)$$

where the vectors q and $\tilde{q}$ have the following form

$$q_r = v_\perp\left[\frac{p}{\xi} J_p(\xi)\cos\psi + iJ_p'(\xi)\sin\psi\right] + v_\parallel \frac{B_r}{B_0} J_p(\xi);$$

$$q_\varphi = v_\perp\left[\frac{p}{\xi} J_p(\xi)\sin\psi - iJ_p'(\xi)\cos\psi\right] + \bar{v}_\varphi J_p(\xi);$$

$$q_z = \left(v_\parallel - \frac{v_\perp^2}{2v_\parallel}\cdot\frac{B_z - B_0}{B_0}\right) J_p(\xi);$$

$$\tilde{q}_r = v_\perp \left[\frac{p}{\xi} J_p(\xi)\cos\psi - iJ'_p(\xi)\sin\psi\right) c_p^{(0)} - \frac{i}{2} v_\parallel J_p(\xi) \sum_l g_l c_p^{(l)} e^{il\alpha z}; \qquad (17)$$ /66

$$\tilde{q}_\varphi = v_\perp \left[\frac{p}{\xi} J_p(\xi)\sin\psi + iJ'_p(\xi)\cos\psi\right] c_p^{(0)} + \bar{v}_\varphi J_p(\xi) c_p^{(0)};$$

$$\tilde{q}_z = v_\parallel J_p(\xi) c_p^{(0)} - \frac{v_\perp^2}{4v_\parallel} J_p(\xi) \sum_l f_l c_p^{(l)} e^{il\alpha z};$$

$$c_p^{(l)} = \left(\omega - p\omega_B - n\alpha s v_\parallel - l\alpha v_\parallel - k_\varphi \bar{v}_\varphi - k_z v_\parallel\right)^{-1}$$

($J_p(\xi)$ is the Bessel function; the sum with respect to l is taken from $-\infty$ to $+\infty$, and it is thus assumed that $g_{-\ell} = -g_\ell$, $f_{-\ell} = j_\ell$ and $f_0 = 0$).

As is well known, the electroconductivity tensor may be employed to obtain the expression for the polarizability vector χ which characterizes the density of the induced charge ρ,

$$\rho = \int dk d\omega e^{i(kr - \omega t)} \chi_\alpha E_\alpha(k, \omega). \qquad (18)$$

The components of the polarizability vector have the following form

$$\chi_\alpha = i\pi \sum_{i,e} e^2 \int dv_\parallel dv_\perp^2 \hat{M}\hat{L} \frac{1}{T} f(v_\parallel, v_\perp, \Psi) \sum_{p=-\infty}^{\infty} J_p(\xi) \tilde{q}_\alpha. \qquad (19)$$

The expressions for the electroconductivity tensor and the polarizability vector may be simplified in the case of electromagnetic oscillations, whose wavelength is much greater than the pinch modulation depth ($\eta_n << 1$). In this case, we must get rid of complex sums and products, since the terms with s = s' = 0 and the operator $\hat{M} = 1$ will make the main contribution.

Plasma Dielectric Constant in a Dielectric Helical Magnetic Field of a Stellarator

The strength components of a nonvortical magnetic field from a helical current winding with the finite step L have the following form (Ref. 6)

$$B_r = B_0 \sum_{n=1}^{\infty} g_n \sin n\theta; \quad B_\varphi = B_0 \sum_{n=1}^{\infty} f_n \cos n\theta; \quad B_z = B_0 - \alpha r B_\varphi, \qquad (20)$$ /67

where B_0 is the strength of a uniform, longitudinal magnetic field; $\theta = \phi - \alpha z$; $\alpha = \frac{2\pi}{L}$; $g_n(r)$ and $f_n(r)$ -- functions of the coordinate r, which are related by the following relationships

$$(rg_n)' = nf_n(1+\alpha^2 r^2);$$
$$(rf_n)' = ng_n. \tag{21}$$

Just as in the case of a pinch field, when deriving the explicit form of the electroconductivity tensor, we investigated a plasma with a small gasokinetic pressure ($\beta << 1$), without a longitudinal current, in a helical magnetic field having a large axial component

$$(f_n \sim g_n \sim \delta \ll 1).$$

Within an accuracy of terms on the order of $\frac{\delta^2}{\omega_B}$, the particle trajectories and velocities may be described by the following expressions

$$\begin{aligned}
r(t) &= r + \sum_{n=1}^{\infty}\frac{g_n}{n\alpha}\left[\cos n\left(\theta+\frac{\bar{v}_\theta t}{r}\right)-\cos n\theta\right]-\frac{v_\perp}{\omega_B}[\sin(\gamma-\\
&\qquad -\omega_B t)-\sin\gamma];\\
\varphi(t) &= \varphi+\frac{\bar{v}_\varphi t}{r}+\sum_{n=1}^{\infty}\frac{f_n}{\alpha nr}\left[\sin n\theta-\sin n\left(\theta+\frac{\bar{v}_\theta t}{r}\right)\right]+\frac{v_\perp}{r\omega_B}\times\\
&\qquad\times[\cos(\gamma-\omega_B t)-\cos\gamma];\\
z(t) &= z+\bar{v}_z t+\frac{v_\perp^2}{2v_\parallel^2}\sum_{n=1}^{\infty}\frac{rf_n}{n}\left[\sin n\left(\theta+\frac{\bar{v}_\theta t}{r}\right)-\sin n\theta\right];\\
v_r(t) &= v_\perp\cos(\gamma-\omega_B t)+v_\parallel\sum_{n=1}^{\infty}g_n\sin n\left(\theta+\frac{\bar{v}_\theta t}{r}\right);\\
r\frac{d\varphi(t)}{dt} &= v_\perp\sin(\gamma-\omega_B t)+v_\parallel\sum_{n=1}^{\infty}f_n\cos n\left(\theta+\frac{\bar{v}_\theta t}{r}\right)+\bar{v}_\varphi;\\
v_z(t) &= \bar{v}_z-\frac{v_\perp^2}{2v_\parallel}\alpha r\sum_{n=1}^{\infty}f_n\cos n\left(\theta+\frac{\bar{v}_\theta t}{r}\right),
\end{aligned} \tag{22}$$

where $\bar{v}_\phi$, $\bar{v}_z$ and $\bar{v}_\theta = \bar{v}_\phi - \alpha r\bar{v}_z$ are the averaged velocity components re- /68
lated to the motion of the center of a Larmor circle along the averaged force line and drift in a nonuniform field:

$$\begin{aligned}
\bar{v}_\varphi &= \alpha r\chi v_\parallel+\frac{v_\perp^2}{r\omega_B}u_1+\frac{v_\parallel^2}{r\omega_B}u_2+\frac{v_\perp^4}{r\omega_B v_\parallel^2}u_3;\\
\bar{v}_z &= v_\parallel-\frac{1}{4}v_\parallel\sum_n\left(f_n^2+g_n^2\right)-\frac{4\left(2v_\parallel^2+v_\perp^2\right)}{r^2\alpha\omega_B}u_3.
\end{aligned} \tag{23}$$

Here χ is the torsion angle of the magnetic force lines:

$$\chi=\frac{1}{2\alpha^2 r^2}\sum_{n=1}^{\infty}\left(f_n^2+g_n^2-\frac{2}{n}g_n f_n+\alpha^2 r^2 f_n^2\right); \tag{24}$$

$$u_1 = \frac{1}{4}\sum_n (f_n^2 + g_n^2 - 2nf_ng_n - 3\alpha^2 r^2 nf_ng_n);$$

$$u_2 = \frac{1}{2}\sum_n (f_n^2 + g_n^2 - 2nf_ng_n - \alpha^2 r^2 nf_ng_n) + \frac{4\pi rp'}{B_0^2}; \tag{25}$$

$$u_3 = -\frac{1}{8}\sum_n \alpha^2 r^2 nf_ng_n.$$

The distribution function F_0 in a slightly nonuniform plasma may also be determined by (10), in which Ψ describes the equation of magnetic surfaces:

$$\Psi = r^2 - 2r\sum_n \frac{g_n}{n\alpha}\cos n\theta. \tag{26}$$

We again select the function f as a Maxwell function, with the density and temperature dependent on the magnetic surfaces Ψ.

As a result of the computations, we obtained the following from the general expression for $\sigma_{\alpha\beta}$ (8)

$$\sigma_{\alpha\beta} = i\pi\sum_{i,\,e} e^2 \int \hat{M}\hat{L}\frac{1}{T} f(v_\perp,\, v_\parallel,\, \Psi) \sum_{p=-\infty}^{\infty} q_\alpha \tilde{q}_\beta dv_\parallel dv_\perp^2, \tag{27}$$

where

$$\hat{M} = \prod_{n=1}^{\infty}\sum_{s,\,s'} J_s(\eta_n) J_{s'}(\eta_n) e^{i(s-s')\left(n\theta+\psi_n+\frac{\pi}{2}\right)};$$

$$\eta_n = \frac{1}{n\alpha}\left\{k_r^2 g_n^2 + k_\varphi^2 f_n^2\left(1 - \frac{k_z}{2k_\varphi}\frac{v_\perp^2}{v_\parallel^2}\alpha r\right)^2\right\}^{1/2};$$

$$\psi_n = \operatorname{arctg}\frac{k_\varphi f_n}{k_r g_n}\left(1 - \frac{k_z v_\perp^2}{2k_\varphi v_\parallel^2}\alpha r\right);$$

$$q_r = v_\perp\left[\frac{p}{\xi} J_p(\xi)\cos\psi + iJ_p'(\xi)\sin\psi\right] + \frac{B_r}{B_0} v_\parallel J_p(\xi);$$

$$q_\varphi = v_\perp\left[\frac{p}{\xi} J_p(\xi)\sin\psi - iJ_p'(\xi)\cos\psi\right] + \bar{v}_\varphi J_p(\xi) + \frac{B_\varphi}{B_0} v_\parallel J_p(\xi);$$

$$q_z = \left(\bar{v}_z - \alpha r\frac{v_\perp^2}{2v_\parallel}\frac{B_\varphi}{B_0}\right) J_p(\xi); \tag{28}$$

/69

$$\tilde{q}_r = v_\perp\left[\frac{p}{\xi} J_p(\xi)\cos\psi - iJ_p'(\xi)\sin\psi\right] c_p^{(0)} - \frac{i}{2} v_\parallel J_p(\xi) \times$$
$$\times \sum_l g_l c_p^{(l)} e^{il\theta};$$

$$\tilde{q}_\varphi = v_\perp \left(\frac{p}{\xi} J_p(\xi) \sin\psi + i J_p'(\xi) \cos\psi \right) c_p^{(0)} + \bar{v}_\varphi J_p(\xi)\, c_p^{(0)} + $$
$$+ \frac{1}{2} v_\parallel J_p(\xi) \sum_l f_l c_p^{(l)} e^{il\theta};$$

$$\tilde{q}_z = \bar{v}_z J_p(\xi)\, c_p^{(0)} - \frac{arv_\perp^2}{4v_\parallel} J_p(\xi) \sum_l f_l c_p^{(l)} e^{il\theta};$$

$$c_p^{(l)} = \left(\omega - k_z \bar{v}_z - k_\varphi \bar{v}_\varphi + \frac{ns\bar{v}_\theta}{r} - \frac{l\bar{v}_\theta}{r} - p\omega_B \right)^{-1}.$$

The quantities k_ϕ, $k_\perp$, ψ, ξ and the operator $\hat{L}$ are determined by the expressions (14); it is assumed that $g_{-\ell} = -g_\ell$, $_{-}f_\ell = f_\ell$ and $f_0 = 0$ in the sums over the index ℓ. Correspondingly, the plasma polarizability vector in a helical magnetic field is

$$\chi_\alpha = i\pi \sum_{i,e} e^2 \int \hat{M} \hat{L} \frac{1}{T} f(v_\parallel, v_\perp, \Psi) \sum_{p=-\infty}^{\infty} J_p(\xi)\, \tilde{q}_\alpha dv_\parallel dv_\perp^2. \qquad (29)$$

Just as in a pinch field, expressions (27) and (29) may be simpli- /70 field, if the oscillation wave length is much greater than the difference between the maximum and minimum radii of the magnetic surface ($\eta_n \ll 1$). We may employ the expressions obtained for $\sigma_{\alpha\beta}$ and χ_α to study the electromagnetic oscillations and plasma stability in pinch magnetic fields and in helical magnetic fields. This will be the subject of future research.

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SECTION II

LINEAR PLASMA OSCILLATIONS /71

KINETIC THEORY OF ELECTROMAGNETIC WAVES IN A CONFINED PLASMA

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Problems of electromagnetic wave propagation in a confined plasma are of considerable interest to studies on methods of plasma heating, acceleration of charged particles, plasma diagnostics, and other possible applications. The hydrodynamic theory of plasma wave guides for slow waves has been studied in great detail [see, for example, the articles (Ref. 1 - 4)]. However, the hydrodynamic theory does not encompass the important phenomena related to the particle thermal motion -- for example, wave damping which is particularly great at small phase velocities. Since the phase velocity of a propagated wave V_Φ is less in the wave guides of slow waves than the speed of light, and since it may be comparable to the mean thermal velocity of electrons v_{Te} or ions v_{Ti}, the necessity of a kinetic examination becomes readily apparent.

On the other hand, the confinement of a plasma leads to a new type of wave -- surface waves -- whose damping, as was shown in (Ref. 5, 6), is proportional to the thermal velocity of plasma electrons for $v_{Te} \ll V_\Phi$. In a nonconfined plasma, where there is no surface wave, the damping of the longitudinal three-dimensional wave is exponentially small (Ref. 7). /72

Formulation of the Problem

Let us investigate the propagation of slow electromagnetic waves in a plasma layer which is 2a thick in one direction, and is not confined in the two other directions. As is known (Ref. 4), waves propagated under these conditions are surface waves when there is no magnetic field.

A self-consistent system of equations describing these processes consists of the Maxwell equations

$$\operatorname{rot} \mathbf{E} = -\frac{1}{c} \cdot \frac{\partial \mathbf{H}}{\partial t}; \tag{1}$$

$$\operatorname{rot} \mathbf{H} = \frac{1}{c} \cdot \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \mathbf{j} \tag{2}$$

and a linearized kinetic equation for the deviation of f_α from the equilibrium distribution function f_{0_α} of the α type of particles (α = i -- ions; α = e -- electrons), in which we shall disregard particle pair collisions

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\frac{\partial f_\alpha}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}, \mathbf{H}]\right)\frac{\partial f_{0\alpha}}{\partial \mathbf{v}} = 0. \tag{3}$$

The standard notation is employed in equations (1) - (3). We shall select the coordinate axes so that the **z** axis coincides with the direction of the wave propagation, and the **x** axis is perpendicular to the layer. The YZ plane is the plane of symmetry.

Equations (1) - (3) must be supplemented by the boundary conditions. Let us assume that $f_\alpha = f_\alpha^+ + f_\alpha^-$, where f_α^+ is the distribution function for $v_x > 0$, and f_α^- -- for $v_x < 0$. We shall select the conditions of the mirror image (Ref. 8) (the final result does not depend quantitatively on the reflection condition) as the boundary conditions for the function $f_\alpha^\pm$

$$f_\alpha^+(\pm a,\ v_x > 0,\ v_z) = f_\alpha^-(\pm a,\ v_x < 0,\ v_z). \tag{4}$$

We obtain the boundary conditions for the fields from the Maxwell equations (1) and (2), integrating them over an infinitely thin layer which encompasses the plasma-vacuum boundary: /73

$$E_z^{pl}(\pm a) = E_z^{vac}(\pm a),\quad H_y^{pl}(\pm a) = H_y^{vac}(\pm a). \tag{5}$$

Dispersion Equation

We can write the dependence of the distribution function f_α on time and the cordinate z for the fields in the form $\exp i(k_3 z - \omega t)$. If $f_{0\alpha}$ is the function of energy, we obtain the following from equation (3) with allowance for the boundary conditions

$$f_\alpha^\pm(x) = \mp \frac{e_\alpha}{m_\alpha v_x}\int\limits_{-a}^{x} d\xi\left[E_z(\xi)\frac{\partial f_{0\alpha}}{\partial v_z} \pm E_z(\xi)\frac{\partial f_{0\alpha}}{\partial v_x}\right]e^{\mp i\gamma(x-\xi)} +$$
$$+ i\frac{e_\alpha}{m_\alpha v_x}\cdot\frac{e^{\mp i\gamma(a+x)}}{\sin 2\gamma a}\int\limits_{-a}^{a} d\xi\left[E_z(\xi)\frac{\partial f_{0\alpha}}{\partial v_z}\cos\gamma(a-\xi) - \right. \tag{6}$$
$$\left. - iE_x(\xi)\frac{\partial f_{0\alpha}}{\partial v_x}\sin\gamma(a-\xi)\right],$$

where

$$\gamma = \frac{k_3 v_z - \omega}{v_x}.$$

In order to compute the currents j_z and j_x, it is necessary to determine the sum and the difference $f_\alpha^+ \pm f_\alpha^-$. Employing the values (6), we obtain

$$f_\alpha^+ + f_\alpha^- = \frac{2ie_\alpha}{m_\alpha v_x}\cdot\frac{\partial f_{0\alpha}}{\partial v_z}\int\limits_{-a}^{a} E_z(\xi)K_1(x,\xi)\,d\xi -$$

$$-\frac{2e_\alpha}{m_\alpha v_x}\cdot\frac{\partial f_{0\alpha}}{\partial v_x}\cdot\frac{1}{\gamma}\cdot\frac{\partial}{\partial x}\int\limits_{-a}^{a} E_x(\xi)K_2(x,\xi)\,d\xi; \tag{7}$$

$$f_\alpha^+ - f_\alpha^- = -\frac{2e_\alpha}{m_\alpha v_x}\cdot\frac{\partial f_{0\alpha}}{\partial v_z}\cdot\frac{1}{\gamma}\cdot\frac{\partial}{\partial x}\int\limits_{-a}^{a} E_z(\xi)K_1(x,\xi)\,d\xi +$$

$$+\frac{2ie_\alpha}{m_\alpha v_x}\cdot\frac{\partial f_{0\alpha}}{\partial v_x}\int\limits_{-a}^{a} E_x(\xi)K_2(x,\xi)\,d\xi, \tag{8}$$

where /74

$$K_1(x,\xi) = \frac{1}{\sin 2\gamma a}\begin{cases}\cos\gamma(\xi-a)\cos\gamma(x+a), & x<\xi,\\ \cos\gamma(x-a)\cos\gamma(\xi+a), & x>\xi;\end{cases} \tag{9}$$

$$K_2(x,\xi) = \frac{1}{\sin 2\gamma a}\begin{cases}\sin\gamma(\xi-a)\sin\gamma(x+a), & x<\xi,\\ \sin\gamma(x-a)\sin\gamma(\xi+a), & x>\xi.\end{cases} \tag{10}$$

The kernels K_1 and K_2, which are expanded in Fourier series, have the following form

$$K_1(x,\xi) = \frac{\gamma}{a}\sum_{n=0}^{\infty}{}'\frac{\cos\alpha_n\xi\cos\alpha_n x}{\gamma^2-\alpha_n^2}; \tag{11}$$

$$K_2(x,\xi) = \frac{\gamma}{a}\sum_{n=0}^{\infty}\frac{\sin\alpha_n\xi\sin\alpha_n x}{\gamma^2-\alpha_n^2}, \tag{12}$$

where $\alpha_n = \frac{n\pi}{a}$; the prime over the sum indicates that the sum term corresponding to $n = 0$ must be multiplied by $\frac{1}{2}$.

Employing the values (7), (8), (11) and (12) and using equation (1) to change from the fields E_x, E_z to the fields E_z, H_y, we may obtain the formulas for determining the currents:

$$j_x = \sum_{n=0}^{\infty}\frac{\sin\alpha_n x}{a}\left\{\alpha_n\int\limits_{-a}^{a}E_z\cos\alpha_n\xi d\xi\sum_{\alpha=i}^{e}\frac{2e_\alpha^2}{m_\alpha}\int\limits_{-\infty}^{\infty}dv_z\int\limits_{0}^{\infty}\frac{dv_x}{\gamma^2-\alpha_n^2}\left(\frac{\partial f_{0\alpha}}{\partial v_z}-\frac{\gamma}{k_3}\cdot\frac{\partial f_{0\alpha}}{\partial v_x}\right)+\right.$$

$$\left.+i\beta\int\limits_{-a}^{a}H_y\sin\alpha_n\xi d\xi\sum_{\alpha=i}^{e}\frac{2e_\alpha^2}{m_\alpha}\int\limits_{-\infty}^{\infty}dv_z\int\limits_{0}^{\infty}\frac{\gamma dv_x}{\gamma^2-\alpha_n^2}\cdot\frac{\partial f_{0\alpha}}{\partial v_x}\right\}; \tag{13}$$

$$j_z = \sum_{n=0}^{\infty}{}' \frac{\cos\alpha_n x}{a} \left\{ \int_{-a}^{a} E_z \cos\alpha_n\xi d\xi \sum_{\alpha=i}^{e} \frac{2ie_\alpha^2}{m_\alpha} \int_{-\infty}^{\infty} \frac{v_z dv_z}{v_x} \int_0^{\infty} \frac{dv_x}{\gamma^2 - \alpha_n^2} \left(\gamma \frac{\partial f_{0\alpha}}{\partial v_z} - \right.\right.$$

$$\left.\left. - \frac{\alpha_n^2}{k_3} \cdot \frac{\partial f_{0\alpha}}{\partial v_x} \right) - \beta\alpha_n \int_{-a}^{a} H_y \sin\alpha_n\xi d\xi \sum_{\alpha=i}^{e} \frac{2e_\alpha^2}{m_\alpha} \int_{-\infty}^{\infty} \frac{v_z dv_z}{v_x} \int_0^{\infty} \frac{dv_x}{\gamma^2 - \alpha_n^2} \cdot \frac{\partial f_{0\alpha}}{\partial v_x} \right\}, \tag{14}$$

where $\beta = \dfrac{\omega}{ck_3}$.

The solution of the Maxwell equations (1) and (2) for determining the fields E_z and H_y leads to the following equations /75

$$\frac{dE_z}{dx} - i\frac{k_3}{\beta}(1-\beta^2) H_y = \frac{4\pi k_3}{\omega} j_x; \tag{15}$$

$$\frac{dH_y}{dx} + i\frac{\omega}{c} E_z = \frac{4\pi}{c} j_z. \tag{16}$$

We may write the solution of the integro-differential system of equations (13) - (16) in the following form

$$E_z(x) = \sum_{n=0}^{\infty}{}' E_{zn} \cos\alpha_n x; \quad H_y(x) = \sum_{n=0}^{\infty} H_{yn} \sin\alpha_n x. \tag{17}$$

We obtained the following value for the Fourier components E_{zn}:

$$E_{zn} = -(-1)^n \frac{2}{a\beta} H_y(a) \frac{A_1\beta + ik_3(1-\beta^2)}{\Delta}, \tag{18}$$

where

$$\Delta = \left(i\frac{\omega}{c} - A_4\right)\left[A_1 + i\frac{k_3}{\beta}(1-\beta^2)\right] + (A_3 - \alpha_n)(A_2 + \alpha_n); \tag{19}$$

$$A_1 = \frac{2i}{c} \sum_{\alpha=i}^{e} \Omega_\alpha^2 \int_{-\infty}^{\infty} dv_z \int_0^{\infty} \frac{\gamma dv_x}{\gamma^2 - \alpha_n^2} \cdot \frac{\partial f_{0\alpha}}{\partial v_x};$$

$$A_2 = 2\frac{k_3\alpha_n}{\omega} \sum_{\alpha=i}^{e} \Omega_\alpha^2 \int_{-\infty}^{\infty} dv_z \int_0^{\infty} \frac{dv_x}{\gamma^2 - \alpha_n^2} \left(\frac{\partial f_{0\alpha}}{\partial v_z} - \frac{\gamma}{k_3} \cdot \frac{\partial f_{0\alpha}}{\partial v_x} \right); \tag{20}$$

$$A_3 = -2\frac{\beta\alpha_n}{c} \sum_{\alpha=i}^{e} \Omega_\alpha^2 \int_{-\infty}^{\infty} v_z dv_z \int_0^{\infty} \frac{dv_x}{(\gamma^2 - \alpha_n^2) v_x} \cdot \frac{\partial f_{0\alpha}}{\partial v_x};$$

$$A_4 = 2\frac{i}{c} \sum_{\alpha=i}^{e} \Omega_\alpha^2 \int_{-\infty}^{\infty} v_z dv_z \int_0^{\infty} \frac{dv_x}{(\gamma^2 - \alpha_n^2) v_x} \left(\gamma \frac{\partial f_{0\alpha}}{\partial v_z} - \frac{\alpha_n^2}{k_3} \cdot \frac{\partial f_{0\alpha}}{\partial v_x} \right).$$

Here $\Omega_\alpha^2 = \dfrac{4\pi e_\alpha^2 \rho_0}{m_\alpha}$; ρ_0 -- equilibrium ion density which equals the equilibrium electron density; $f_{0\alpha}$ is the density normalized to unity ($\int f_{0\alpha}\, dV = 1$).

Selecting Maxwell distributions as the equilibrium distribution functions $f_{0\alpha}$, we may write the equation for the plasma impedance /76

$$\frac{E_z(a)}{iH_y(a)} = -\frac{2}{a}\cdot\frac{k_3}{\beta}\sum_{n=0}^{\infty}{}' \frac{1}{\Delta}\left[1-\beta^2\left(1-\sum_{\alpha=i}^{e}\frac{\Omega_\alpha^2}{\omega^2}Q_{1\alpha}\right)\right]. \tag{21}$$

We thus have

$$\begin{aligned}\Delta = -k_3^2\left[1-\sum_{\alpha=i}^{e}\frac{\Omega_\alpha^2}{\omega^2}\left(Q_{2\alpha}+\frac{\alpha_n^2}{k_3^2}Q_{3\alpha}\right)\right]\left[1-\beta^2\left(1-\sum_{\alpha=i}^{e}\frac{\Omega_\alpha^2}{\omega^2}Q_{1\alpha}\right)\right]- \\ -\alpha_n^2\left(1-\beta^2\sum_{\alpha=i}^{e}\frac{\Omega_\alpha^2}{\omega^2}Q_{3\alpha}\right)\left[1-\sum_{\alpha=i}^{e}\frac{\Omega_\alpha^2}{\omega^2}(Q_{1\alpha}+Q_{3\alpha})\right];\end{aligned} \tag{22}$$

$$\begin{aligned}Q_{1\alpha} &= \frac{2}{\pi}\sigma_\alpha\int_{-\infty}^{\infty} dz\,(\sigma_\alpha-bz)\,e^{-z^2}\left[(\sigma_\alpha-bz)^2 I_\alpha(z)-\sqrt{\pi}\right];\\ Q_{2\alpha} &= \frac{2}{\pi}\sigma_\alpha\int_{-\infty}^{\infty} dz z^2(\sigma_\alpha-bz)\,e^{-z^2}I_\alpha(z);\\ Q_{3\alpha} &= \frac{2}{\pi}\sigma_\alpha b\int_{-\infty}^{\infty} dz z(\sigma_\alpha-bz)^2 e^{-z^2}I_\alpha(z)\end{aligned} \tag{23}$$

$\left(\sigma_\alpha = \dfrac{\omega}{\alpha_n v_{T\alpha}}\right.$; $v_{T\alpha} = \sqrt{\dfrac{2T_\alpha}{m_\alpha}}$ -- the mean thermal velocity of α type of particles;

$\left. b = \dfrac{k_3}{\alpha_n};\ I_\alpha(z) = \displaystyle\int_{-\infty}^{\infty}\frac{e^{-x^2}dx}{(\sigma_\alpha-bz)^2-x^2}\right).$

Equating the plasma impedance (21) to the vacuum impedance

$$\frac{E_z(a)}{iH_y(a)} = -\frac{c}{\omega}k_3(1-\beta^2)^{\frac{1}{2}}, \tag{24}$$

we obtain the dispersion equation

$$\frac{2}{a} k_3 \sum_{n=0}^{\infty}{}' \frac{1}{\Delta} \left[1 - \beta^2 \left(1 - \sum_{\alpha=i}^{e} \frac{\Omega_\alpha^2}{\omega^2} Q_{1\alpha} \right) \right] = (1 - \beta^2)^{\frac{1}{2}}. \tag{25}$$

The sum (25) consists of the real and imaginary parts, since each Q contains the real part which equals the main value, and the imaginary part which is proportional to the residue of the integral $I_\alpha(z)$. /77

High Frequency Oscillations

Let us set $v_{Ti} = 0$, $\frac{k_3 v_{Te}}{\omega} \ll 1$, $m_i \to \infty$.

We shall employ the following notation

$$P_1 = \frac{2k_3}{a} \mathrm{Re} \sum_{n=0}^{\infty}{}' \frac{1}{\Delta} \left[1 - \beta^2 \left(1 - \frac{\Omega_e^2}{\omega^2} Q_1 \right) \right]; \tag{26}$$

$$P_2 = \frac{2k_3}{a} \mathrm{Im} \sum_{n=0}^{\infty}{}' \frac{1}{\Delta} \left[1 - \beta^2 \left(1 - \frac{\Omega_e^2}{\omega^2} Q_1 \right) \right]. \tag{27}$$

Since α_n are evenly included in the sum (26), we may change from the sum to the integral (Ref. 8)

$$P_1 = \frac{i}{\pi} k_3 \int_C \frac{\mathrm{ctg}\, qa}{\Delta(q)} \left[1 - \beta^2 \left(1 - \frac{\Omega_e^2}{\omega^2} Q_1(q) \right) \right] dq, \tag{28}$$

in which integration over the contour C is performed from $-\infty$ to $+\infty$, passing around the singular points $q = \frac{n\pi}{a}$ from above. Let us deform the integration contour, and let us enclose it in the upper half-plane. Let us set $q_0 = ik_\perp$ -- the value of q at which $\Delta(q) = 0$. We then have

$$P_1 = \frac{k_3}{k_\perp} \cdot \frac{1 - \beta^2 \varepsilon}{-\varepsilon} \mathrm{cth}\, k_\perp a, \tag{29}$$

since the integrals Q_j have the following principal values Q_j'' (j = 1, 2, 3) when the electron pressure is disregarded:

$$Q_1'' = Q_2'' = 1,\ Q_3'' = 0 \left(\varepsilon = 1 - \frac{\Omega_e^2}{\omega^2},\ k_\perp^2 = k_3^2 (1 - \beta^2 \varepsilon) \right).$$

When computing the sum P_2, we should note that -- since $\sigma_e = b \frac{\omega}{k_3 v_{Te}} \gg b$ -- for small n, just as for $\sigma_e \gg 1$, the integral residue I(z) is exponentially small. Therefore, components with n which is larger than a certain n_0 (i.e., shortwave components of Fourier expansion), which may be determined from the condition $\sigma_e \sim 1$, make the main contribution to the sum P_2. For such n, $b^2 \ll 1$, and Q_j have the following values (we shall omit the index e): /78

$$Q_1 = 2\sigma^2\left[-1+\sigma e^{-\sigma^2}\left(2\int_0^\sigma e^{t^2}dt - i\sqrt{\pi}\right)\right];$$
$$Q_2 = \sigma e^{-\sigma^2}\left(2\int_0^\sigma e^{t^2}dt - i\sqrt{\pi}\right); \tag{30}$$
$$Q_3 = 2\sigma b^2\left[-\sigma+\left(\sigma^2-\frac{1}{2}\right)e^{-\sigma^2}\left(2\int_0^\sigma e^{t^2}dt - i\sqrt{\pi}\right)\right].$$

Since $Q_1 \sim \sigma^2$, $Q_2 \sim \sigma$, $\frac{Q_3}{b^2} \sim \sigma$, in the case of $b^2 \ll \sigma^2$, $b^2 \ll 1$ for Δ, which is determined by equation (22), we obtain the following expression

$$\Delta = -\alpha_n^2\left(1-\frac{\Omega_e^2}{\omega^2}Q_1\right). \tag{31}$$

Let us employ the following notation: $Q_1'' = \mathrm{Re}Q_1$, $Q_1' = \mathrm{Im}\,Q_1$. We then have

$$P_2 = -\frac{2}{a}k_3\frac{\Omega_e^2}{\omega^2}\cdot\frac{v_{Te}^2}{\omega^2}\sum_{n>n_0}^{\infty}\frac{\sigma^2 Q_1'}{\left(1-\frac{\Omega_e^2}{\omega^2}Q_1''\right)^2+\left(\frac{\Omega_e^2}{\omega^2}Q_1'\right)^2}. \tag{32}$$

Since $n_0 \gg 1$, we may change from the sum to the integral

$$P_2 = -\frac{4}{\sqrt{\pi}}\cdot\frac{\Omega_e^2}{\omega^2}\cdot\frac{k_3 v_{Te}}{\omega}\int_{\sigma_0}^{\delta}\frac{\sigma^3 e^{-\sigma^2}}{\left(1-\frac{\Omega_e^2}{\omega^2}Q_1''\right)^2+\left(\frac{\Omega_e^2}{\omega^2}Q_1'\right)^2}. \tag{33}$$

We may disregard the quantity $Q_1'^2$ in the denominator of the integral, and for Q_1'' we may employ its value in the case of $\sigma \sim 1$. In addition, making an exponentially small error, we may multiply the lower integration limit by ∞. We finally obtain

$$P_2 = \frac{2}{\sqrt{\pi}}\cdot\frac{1-\bar{\varepsilon}}{\bar{\varepsilon}^2}\cdot\frac{k_3 v_{Te}}{\omega};\quad \bar{\varepsilon} = 1-\frac{\Omega_e}{\omega_2}Q_1''/_{\sigma\sim 1}. \tag{34}$$

The dispersion equation (25) now assumes the following form

$$\frac{(1-\beta^2\bar{\varepsilon})^{\frac{1}{2}}}{-\bar{\varepsilon}}\,\mathrm{cth}\,k_\perp a + i\frac{2}{\sqrt{\pi}}\cdot\frac{1-\bar{\varepsilon}}{\bar{\varepsilon}^2}\cdot\frac{k_3 v_{Te}}{\omega} = (1-\beta^2)^{\frac{1}{2}}. \tag{35}$$

Let us assume that $k_3 \to k_3' = k_3 + i\delta$, $\delta \ll k_3$. Then the real part of equation (35) produces a relationship between the phase velocity β and /79 the frequency of the propagated wave ω:

$$\frac{(1-\beta^2\varepsilon)^{\frac{1}{2}}}{-\varepsilon}\operatorname{cth}k_\perp a=(1-\beta^2)^{\frac{1}{2}},\tag{36}$$

and the imaginary part produces damping:

$$\delta=\frac{2}{\sqrt{\pi}}\cdot\frac{k_3^2 v_{Te}}{\omega}\cdot\frac{(1-\beta^2)(1-\varepsilon)}{\frac{\beta^2\varepsilon^2(1-\varepsilon)}{1-\beta^2\varepsilon}+k_\perp a\frac{\operatorname{cth}k_\perp a}{\operatorname{sh}^2 k_\perp a}}\cdot\frac{\varepsilon^2}{\bar{\varepsilon}^2}.\tag{37}$$

Formula (37) can be considerably simplified in the case of large ($k_\perp a \gg 1$) and small ($k_\perp a \ll 1$) layer thickness. We have excluded β^2 from equation (36), and shall substitute it in (37). In the case of $k_\perp a \gg 1$, we obtain

$$\delta=\frac{2}{\sqrt{\pi}}\cdot\frac{k^2 v_{Te}}{\omega}\cdot\frac{\sqrt{|\varepsilon|}}{(1-|\varepsilon|)^2}\cdot\frac{\varepsilon^2}{\bar{\varepsilon}^2},\tag{38}$$

where $k=\frac{\omega}{c}$. The damping determined by this formula coincides with the damping found in (Ref. 5, 6) for a reflection coefficient of $P = 1$. In the case of $k_\perp a \ll 1$, we have

$$\delta=\frac{2}{\sqrt{\pi}}k v_{Te}\frac{1-\varepsilon}{\varepsilon^2}\cdot\frac{(c^2+\varepsilon^2a^2\omega^2)^{\frac{1}{2}}}{\omega^2a^2\cdot\bar{\varepsilon}^2}.\tag{39}$$

It may thus be seen that wave damping is increased when there is a decrease in the plasma layer thickness.

Ion-Sound Waves ($v_{Te} \gg V_\Phi \gg v_{Ti}$)

The dispersion equation (25) may be considerably simplified, if we set the speed of light $c = \infty$. In this case, we have

$$\Delta=-k_3^2\left\{1+\frac{1}{b^2}\left[1-\sum_{\alpha=i}^{e}\frac{\Omega_\alpha^2}{\omega^2}(Q_1+b^2Q_2+2Q_3)_\alpha\right]\right\}.\tag{40}$$

We shall employ the following notation: $\Delta''=\operatorname{Re}\frac{\Delta}{-k_3^2}$, $\Delta'=\operatorname{Im}\frac{\Delta}{-k_3^2}$. /80

Then the real P_3 and the imaginary P_4 components in the sum (25) have the following form

$$P_3=-\frac{2}{ak_3}\sum_{n=0}^{\infty}{}'\frac{\Delta''}{\Delta''^2+\Delta'^2};\tag{41}$$

$$P_4=\frac{2}{ak_3}\sum_{n=0}^{\infty}\frac{\Delta'}{\Delta''^2+\Delta'^2}.\tag{42}$$

We shall employ the same procedure in computing the sum P_3 as for computing P_1, but we shall take the fact into account that in the case of $\frac{\omega}{k_3 v_{Te}} \ll 1$ we have

$$\mathrm{Re}\left[Q_1(q) + \frac{k_3^2}{q^2} Q_2(q) + 2Q_3(q)\right]_e = -2\frac{\omega^2}{q^2 v_{Te}^2}. \qquad (43)$$

We obtain

$$P_3 = \frac{k_3}{\varkappa} \cdot \frac{\mathrm{cth}\,\varkappa a}{-\varepsilon_i}, \qquad (44)$$

where

$$\varkappa^2 = k_3^2(1+\xi); \quad \xi = \frac{2\Omega_e^2}{k_3^2 \varepsilon_i v_{Te}^2}; \quad \varepsilon_i = 1 - \frac{\Omega_i^2}{\omega^2}; \quad \mu = \frac{m_e}{m_i}.$$

Employing the integrals (23), we find

$$\mathrm{Im}(Q_1 + b^2 Q_2 + 2Q_3)_\alpha = -\frac{2\sqrt{\pi}\sigma_\alpha^3}{(1+b^2)^{\frac{1}{2}}} \exp\left(-\frac{\sigma_\alpha^2}{1+b^2}\right). \qquad (45)$$

Since $\frac{\sigma_e^2}{1+b^2} \ll 1$ for any n, we then have

$$\Delta' = \frac{2\sqrt{\pi}\, b}{(1+b^2)^{\frac{1}{2}}} \cdot \frac{\Omega_i^2}{\omega^2}\left[\left(\frac{\omega}{k_3 v_{Ti}}\right)^3 e^{-\frac{\sigma_i^2}{1+b^2}} + \frac{1}{\mu}\left(\frac{\omega}{k_3 v_{Te}}\right)^3\right]. \qquad (46)$$

Consequently, both electrons and ions make a contribution to the wave damping. The ion component P_{4i} of the sum P_4 has the same form as P_2 with a replacement of the indices e → i:

$$P_{4i} = \frac{2}{\sqrt{\pi}} \cdot \frac{1-\varepsilon_i}{\varepsilon_i^2} \cdot \frac{k_3 v_{Ti}}{\omega}. \qquad (47)$$

In computing the electron component P_{4e} of the sum $\dot{P}_4$, and in computing /81 $\tilde{P}_{4i}$, we shall set $\Delta''^2 \gg \Delta'^2$. We then have

$$P_{4e} = \frac{4\sqrt{\pi}}{ak_3} \cdot \frac{1-\varepsilon_i}{\varepsilon_i^2} \cdot \frac{1}{\mu}\left(\frac{\omega}{k_3 v_{Te}}\right)^3 \sum_{n=0}^{\infty}{}' \frac{\left(1+\frac{n^2\pi^2}{k_3^2 a^2}\right)^{-\frac{1}{2}}}{\left(1+\xi+\frac{n^2\pi^2}{k_3^2 a^2}\right)^2}. \qquad (48)$$

Thus, the dispersion equation (25) has the following form for ion-sound waves

$$\frac{k_3}{\varkappa} \cdot \frac{\mathrm{cth}\,\varkappa a}{-\varepsilon_i} + i(P_{4i} + P_{4e}) = 1. \qquad (49)$$

We may thus find the expression for the spatial damping decrement

$$\delta = k_3(1+\xi)\frac{-\varepsilon_i(P_{4i}+P_{4e})}{\xi\varepsilon_i + \frac{k_3 a}{\mathrm{sh}^2\varkappa a}}. \tag{50}$$

In a nonconfined plasma, where there is no surface wave, the damping of a three-dimensional ion-sound wave is determined by Cherenkov absorption of the wave energy by plasma electrons, and the ion contribution to the damping is exponentially small. As may be seen from formulas (47) - (50), plasma ions also make a great contribution to the damping for a surface ion-sound wave. The physics here is the same as for high frequency oscillations -- the Cherenkov absorption of the wave energy by plasma ions is particularly significant for the short wave components of Fourier expansion of the surface wave.

Let us study equations (49) and (50) in special, different cases. If $k_3 a(1+\xi)^{1/2} \gg 1$, then we obtain from equation (49)

$$k_3^2 = \frac{2\Omega_e^2}{v_{Te}^2}\cdot\frac{\varepsilon_i}{1-\varepsilon_i^2}; \tag{51}$$

$$\delta = \frac{4}{\sqrt{\pi}}\cdot\frac{\Omega_e^2}{v_{Te}^2}\cdot\frac{1-\varepsilon_i}{|\varepsilon_i|\left(1-\varepsilon_i^2\right)^2}\left[\frac{v_{Ti}}{\omega}\frac{\varepsilon_i^2}{\overline{\varepsilon_i^2}} + \frac{\omega^3 v_{Te}}{4\mu\Omega_e^4}\left(1-\varepsilon_i^2\right)^2|\varepsilon_i|\right], \tag{52}$$

since for large $k_3 a$ the sum included in P_{4e} approximately equals $\frac{k_3 a}{2\pi}$.

It can be seen from formula (52) that the ions make a particularly large contribution to the damping, if $\varepsilon_i \to -1$. If $k_3 a(1+\xi)^{1/2} \ll 1$, then /82

$$k_3 = \frac{1}{-2\varepsilon_i a}\left[1+\left(1-8\varepsilon_i\frac{\Omega_e^2 a^2}{v_{Te}^2}\right)^{\frac{1}{2}}\right]; \tag{53}$$

$$\delta = k_3\frac{1+\xi}{1-\xi}(P_{4i}+P_{4e}). \tag{54}$$

The following limiting cases are possible: (1) At a large plasma electron temperature or a small layer thickness, when $-8\varepsilon_i\frac{\Omega_e^2 a^2}{v_{Te}^2} \ll 1$ $(|\xi| \ll 1)$, the damping decrement is

$$\delta = \frac{2}{\sqrt{\pi}}\cdot\frac{1-\varepsilon_i}{a^2\varepsilon_i^4}\left[\frac{v_{Ti}}{\omega}\cdot\frac{\varepsilon_i^2}{\overline{\varepsilon_i^2}} + \frac{\pi\omega}{\mu v_{Te}^3}a^4|\varepsilon_i|^5\right] \tag{55}$$

(in the case of $k_3 a \ll 1$ the sum included in P_{4e} approximately equals $\frac{1}{2}$); (2) For a large plasma layer thickness or an electron temperature which is not too high when $-8\varepsilon_i\frac{\Omega_e^2 a^2}{v_{Te}^2} \gg 1$ $(\xi \approx -1)$,

$$\delta = \sqrt{\frac{2}{\pi}} \frac{1-\varepsilon_i}{|\varepsilon_i|^{\frac{7}{2}}} \cdot \frac{\Omega_e}{v_{Te}} \left[\frac{v_{Ti}}{a\omega} \cdot \frac{\varepsilon_i^2}{\varepsilon_i^2} + \frac{\pi\omega^3 |\varepsilon_i|^{\frac{7}{2}}}{\sqrt{8}\mu\Omega_e^3} \right]. \tag{56}$$

We cannot assume that a → ∞ here, due to the condition of the initial expansion.

As may be seen from formulas (52), (55) and (56), the damping decrements of a surface ion-sound wave are large in different cases. Plasma ions make a significant contribution, and sometimes the main contribution, to the damping. The wave phase velocity is decreased with a decrease in the layer thickness; therefore, the ion contribution to the damping increases, while the electron contribution decreases.

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KINETIC THEORY OF A SURFACE WAVE IN A PLASMA WAVE GUIDE

M. F. Gorbatenko, V. I. Kurilko

As was shown in (Ref. 1), when a surface wave is propagated along a plane boundary of half-space occupied by a plasma having a temperature which is different from zero, Cherenkov absorption of the wave energy by thermal plasma electrons takes place. This leads to damping of this wave even when there are no collisions. In contrast to the damping of a longitudinal wave in an unconfined plasma, in this case for small thermal velocities the damping coefficient is proportional to the thermal velocity of plasma electrons.

We investigated this phenomenon for the case of a plasma wave guide produced by a plasma layer having a finite thickness (2l, |x| < l). Since the electrons moving at a thermal velocity are successively reflected from both walls of the wave guide, it was not known previously that in this case the absorption investigated in (Ref. 1) will not decrease considerably.

The initial system of equations consists of the kinetic equation for a high frequency addition to the distribution function and of a Maxwell equation:

$$\left.\begin{aligned} &\frac{\partial f}{\partial t}+\vec{v}\nabla_{\vec{r}}f-\frac{e}{m}\vec{E}\nabla_{\vec{v}}f_0=0\\ &\operatorname{rot}\vec{H}=\frac{4\pi}{c}\vec{J}+\frac{1}{c}\cdot\frac{\partial\vec{E}}{\partial t}\\ &\operatorname{rot}\vec{E}=-\frac{1}{c}\cdot\frac{\partial\vec{H}}{\partial t}\\ &\vec{J}=-e\int\limits_{-\infty}^{+\infty}\vec{v}f\,d\vec{v}\end{aligned}\right\}. \tag{1}$$

It is assumed that the equilibrium distribution function is a Maxwell distribution: $f_0=\frac{n_0}{\pi v_T^2}\exp\left[-\frac{v_z^2+v_x^2}{v_T^2}\right]$ (the z axis passes along the direction in which a surface wave is propagated). /84

Let us write the solution for system (1) in the following form

$$f(x,\ z,\ t)=\varphi(x)\exp[i(k_3z-\omega t)],\ \vec{E}(x,\ z,\ t)=$$
$$=\vec{E}(x)\exp[i(k_3z-\omega t)].$$

We then obtain the following equation for the distribution functions of particles moving in positive (+) and negative (−) directions along the x axis

$$-i\omega^*\varphi^\pm \pm v_x \frac{\partial\varphi^\pm}{\partial x} + \frac{2ef_0}{mv_T^2}(v_zE_z \pm v_xE_x) = 0. \qquad (2)$$

We shall employ the following expression as the boundary conditions for the distribution function $\phi^\pm(x)$ in the planes $x = \pm 1$ of a plasma layer

$$\begin{aligned}\varphi^+(x=-l) &= p\varphi^-(x=-l),\\ \varphi^-(x=l) &= p\varphi^+(x=l).\end{aligned} \quad (0 \leqslant p \leqslant 1) \qquad (3)$$

The solution for equation (2) which satisfies the boundary conditions has the following form

$$\begin{aligned}\varphi^+(x) = &-\frac{2ef_0}{mv_x v_T^2\Delta}\left\{\int_{-l}^{x}(v_zE_z + v_xE_x)\exp\left[-\frac{i\omega^*}{v_x}(x'-x+2l)\right]dx' + \right.\\ &+ p\int_{-l}^{l}(v_zE_z - v_xE_x)\exp\left[\frac{i\omega^*}{v_x}(x'+x)\right]dx' - p^2\int_{l}^{x}(v_zE_z + v_xE_x)\times\\ &\left.\times\exp\left[-\frac{i\omega^*}{v_x}(x'-x-2l)\right]dx'\right\};\end{aligned} \qquad (4)$$

$$\begin{aligned}\varphi^-(x) = &\frac{2ef_0}{mv_x v_T^2\Delta}\left\{\int_{l}^{x}(v_zE_z - v_xE_x)\exp\left[\frac{i\omega^*}{v_x}(x'-x-2l)\right]dx' - \right.\\ &- p\int_{-l}^{l}(v_zE_z + v_xE_x)\exp\left[-\frac{i\omega^*}{v_x}(x'+x)\right]dx' - p^2\int_{-l}^{x}(v_zE_z - v_xE_x)\times\\ &\left.\times\exp\left[\frac{i\omega^*}{v_x}(x'-x+2l)\right]dx'\right\},\end{aligned}$$

where /85

$$\Delta = e^{-\frac{2i\omega^*}{v_x}l} - p^2 e^{\frac{2i\omega^*}{v_x}l};\ \omega^* = \omega - k_3v_z;\ \omega_0^2 = \frac{4\pi e^2 n_0}{m},$$

n_0 -- plasma electron density.

By substituting system (4) in the expression for the current, and by substituting the latter in the Maxwell equation, we obtain a system of two integro-differential equations for the E_z and E_x fields. In the general case, this system is cumbersome, and it is difficult to obtain an analytical solution for it. However, in the case of slight thermal scatter ($v_T \to 0$), a solution may be found for the system. In actuality, in this case, as may be readily shown, current corrections due to the electron thermal motion decrease with an increase in the distance from the surface of the plasma layer proportionally to

$$\exp\left[-R^{2/3}\,|\,1-\eta\,|^{2/3}\right]\left(R=\frac{\omega l}{v_T},\quad \eta=\frac{x}{l}\right).$$

Since there are significant thermal scatter phenomena only in a layer having a thickness on the order of $\frac{v_T}{\omega}$ close to the plasma surface, they may be taken into account by means of effective conditions which the fields determined according to the Maxwell equation for "a cold" plasma must satisfy.

The effective boundary conditions follow from the Maxwell equations, in which the currents are expressed by the fields by means of system (4). For purposes of definition, let us investigate a wave in which the longitudinal field component is symmetrical with respect to the plane $x = 0$ $[E_z,(-x) = E_z(x),\ H_y(-x) = -H_y,(x)]$ (Ref. 2). As would be expected, the currents J_x and J_z -- according to system (4) -- retain the symmetry of the corresponding fields. Therefore, it is sufficient to obtain the boundary conditions for one of the plasma layer surfaces; they will be satisfied on the second surface due to the field symmetry. Let us find the boundary conditions which must be satisfied by the tangential field components on the $x = 1$ surface. Integrating equations containing J_x and J_z, over an infinitely thin layer $\varepsilon(\frac{v_T}{\omega} \ll \varepsilon \ll \delta,\ 1$, where ε -- layer thickenss, δ -- depth of skin-layer), we obtain /86

$$\begin{aligned} H_y^e - H_y^i &= \frac{4\pi l}{c} J_z^* = \frac{4\pi}{c}\int_{l-\varepsilon}^{l+\varepsilon} J_z(\eta)\,d\eta; \\ E_z^e - E_z^i &= \frac{4\pi k_3 l}{\omega} J_x^* = \frac{4\pi k_3}{\omega}\int_{l-\varepsilon}^{l+\varepsilon} J_x(\eta)\,d\eta. \end{aligned} \tag{5}$$

When the integrals in equations (5) are computed, it is advantageous first to perform integration over η:

$$\frac{2\pi\sqrt{\pi}\omega}{\omega_0^2} J_x^* = -i\int_0^\infty \frac{\xi^2 d\xi}{\Delta}\exp\left[-\xi^2-\frac{2iR}{\xi}\right]\left\{\int_{1-\varepsilon}^{1} E_x(\eta')\times\right.$$

$$\times\exp\left[-\frac{iR}{\xi}(1-\varepsilon-\eta')\right]d\eta' -$$

$$\left.-2\int_{1-\varepsilon}^{1+\varepsilon} E_x(\eta')\,d\eta'\right\} - ip^2\int_0^\infty\frac{\xi^2 d\xi}{\Delta}\exp\left[-\xi^2+\frac{2iR}{\xi}\right]\left\{-\int_{1-\varepsilon}^{1} E_x(\eta')\times\right.$$

$$\times\exp\left[\frac{iR}{\xi}(1-\varepsilon-\eta')\right]d\eta' + 2\left.\int_{1-\varepsilon}^{1+\varepsilon} E_x(\eta')\,d\eta'\right\} - ip\int_0^\infty\frac{\xi^2 d\xi}{\Delta}\times$$

$$\times \exp(-\xi^2)\left\{\int_{-1}^{1} E_x(\eta')\exp\left[-\frac{iR}{\xi}(1+\eta')\right]d\eta' - \right.$$

$$\left. - \int_{-1}^{1} E_x(\eta')\exp\left[\frac{iR}{\xi}(1+\eta')\right]d\eta'\right\}; \qquad (6)$$ /87

$$\frac{4\pi\sqrt{\pi}\omega}{\omega_0^2} J_z^* = i\int_0^\infty \frac{d\xi}{\Delta}\exp\left[-\xi^2 - \frac{2iR}{\xi}\right]\int_1^{1-\varepsilon} E_z(\eta')\exp\left[-\frac{iR}{\xi}\times\right.$$

$$\left.\times(1-\varepsilon-\eta')\right]d\eta' - ip^2\int_0^\infty \frac{d\xi}{\Delta}\exp\left[-\xi^2 + \frac{2iR}{\xi}\right]\int_1^{1-\varepsilon} E_z(\eta')\times$$

$$\times \exp\left[\frac{iR}{\xi}(1-\varepsilon-\eta')\right]d\eta' + ip\int_0^\infty \frac{d\xi}{\Delta}\exp(-\xi^2)\left\{\int_{-1}^{1} E_z(\eta')\times\right.$$

$$\left.\times \exp\left[-\frac{iR}{\xi}(1+\eta')\right]d\eta' - \int_{-1}^{1} E_z(\eta')\exp\left[\frac{iR}{\xi}(1+\eta')\right]d\eta'\right\},$$

where only terms making the largest contribution in the case of $v_T \to 0$ are retained. Performing integration over η' in equations (6), we find

$$\frac{2\pi\sqrt{\pi}\omega}{\omega_0^2} J_x^* = \int_0^\infty \exp(-\xi^2)\xi^2 d\xi\left\{\frac{\xi E_x^i(1)}{R}\cdot\frac{e^{-\frac{2iR}{\xi}} + p^2 e^{\frac{2iR}{\xi}}}{e^{-\frac{2iR}{\xi}} - p^2 e^{\frac{2iR}{\xi}}} + \right.$$

$$\left. + \frac{2\left(E_z^e - E_z^i\right)}{k_3 e}\right\} + \frac{p}{R}\int_0^\infty E_x^i(1)\exp(-\xi^2)\xi^3 d\xi\,\frac{\left[e^{-\frac{iR}{\xi}} + e^{\frac{iR}{\xi}}\right]^2}{e^{-\frac{2iR}{\xi}} - p^2 e^{\frac{2iR}{\xi}}};$$

$$\frac{4\pi\sqrt{\pi}\omega}{\omega_0^2} J_z^* = \frac{E_z^i(1)}{R}\int_0^\infty \exp(-\xi^2)\xi d\xi\,\frac{e^{-\frac{2iR}{\xi}} + p^2 e^{\frac{2iR}{\xi}}}{e^{-\frac{2iR}{\xi}} - p^2 e^{\frac{2iR}{\xi}}} - \qquad (7)$$

$$-\frac{p}{R}\int_0^\infty E_z^i(1)\exp(-\xi^2)\xi d\xi\,\frac{\left[e^{-\frac{iR}{\xi}} + e^{\frac{iR}{\xi}}\right]^2}{e^{-\frac{2iR}{\xi}} - p^2 e^{\frac{2iR}{\xi}}}.$$

Since J_x^* and J_z^* in equations (7) are proportional to v_T, when the integrals are computed with respect to $\xi = \frac{v_x}{v_T}$ in these equations, it is sufficient to confine oneself to the zero-th approximation with respect to v_T (i.e., to strive to zero in the v_T intervals). Taking the fact into account that $\mathrm{Im}\omega > 0$, we finally obtain

$$J_z^* = \frac{\omega_0^2 (1-p) E_z^i(1);}{8\pi \sqrt{\pi}\omega R} \qquad |\varepsilon_3| \gg 1$$
$$J_x^* = \frac{\omega_0^2 \left(E_z^e - E_z^i\right)}{4\pi\omega k_3 l} + \frac{\omega_0^2 (1+p)}{4\pi\sqrt{\pi}\omega R} E_x^i(1). \tag{8}$$

Thus, the effective boundary conditions taking into account the thermal motion of plasma electrons have the following form /88

$$H_y^e - H_y^i = \frac{kl\omega_0^2(1-p)}{2\sqrt{\pi}\omega^2 R} E_z^i\big|_{\eta=1};$$
$$\varepsilon_3\left(E_z^e - E_z^i\right) - \frac{kl\omega_0^2(1+p)}{\sqrt{\pi}\,\omega^2 R} H_y^i + \frac{i\omega_0^2(1+p)}{\sqrt{\pi}\omega^2 R} E_z'^i\big|_{\eta=1} = 0. \tag{9}$$

Substituting the solution of the Maxwell equations for a plasma ($|\eta| < 1$) in a vacuum ($|\eta| > 1$) in these boundary conditions

$$E_z^i(\eta) = A\,\mathrm{ch}\,\tau l\eta;\quad E_z''(\eta) = \tau l A\,\mathrm{sh}\,\tau l\eta;\quad H_y^i(\eta) = -\frac{ik\varepsilon_3}{\tau} A\,\mathrm{sh}\,\tau l\eta,$$

where

$$\tau^2 = k_3^2 - \varepsilon_3 k^2;\ \varepsilon_3 = 1 - \frac{\omega_0^2}{\omega^2};\quad k = \frac{\omega}{c};$$
$$E_z^e(\eta) = Be^{-\tilde{\varkappa} l\eta},\ E_z'^e(\eta) = -\tilde{\varkappa} lBe^{-\tilde{\varkappa} l\eta},\ H_y^e(\eta) = \frac{ik}{\tilde{\varkappa}} Be^{-\tilde{\varkappa} l\eta}; \tag{10}$$
$$\tilde{\varkappa}^2 = k_3^2 - k^2,$$

we obtain the dispersion equation

$$\varepsilon_3\left[\tau + \varepsilon_3\tilde{\varkappa}\,\mathrm{th}\,\tau l\right] = \frac{i\omega_0^2 v_T}{\sqrt{\pi}\omega^3}\left[(1+p)\,k_3^2\,\mathrm{th}\,\tau l - \frac{\varepsilon_3\tilde{\varkappa}\tau}{2}(1-p)\right]. \tag{11}$$

In the case of $l \to \infty$, this equation changes into an equation for half-space (Ref. 1).

Thus, in the case of a plasma layer having a finite thickness, the surface wave damping is proportional to v_T. However, the wave damping increases with a decrease in the layer thickness, because the wave phase velocity decreases:

$$\Delta k_3 = -\frac{i\omega_0^2 v_T}{\sqrt{\pi} k_{30}\varepsilon_3^3 l^3\omega^3}\left[(1+p)\,k_{30}^2\, l^2 + \frac{(1-p)}{2}\right];\ |\varepsilon_3| \gg 1,\ \varepsilon_3 < 0; \tag{12}$$
$$k_{30}^2 = k^2 + \frac{1}{\varepsilon_3^2 l^2};\ k_3^2 l^2 \ll 1.$$

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SINGULARITIES OF AN ELECTROMAGNETIC FIELD IN A NONUNIFORM, MAGNETOACTIVE PLASMA /89

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As is well known, in an isotropic, nonuniform transmittant medium there is a sharp increase in the electric field strength of an electromagnetic wave at the point where the dielectric constant vanishes. This increase is caused by plasma resonance. This phenomenon, which has been called "inflation" of the field has been studied in several articles. It is pointed out in (Ref. 1) that the "inflation" must occur in a magnetoactive medium at the point where the refractive index for the wave becomes infinite. The phenomenon of field "inflation" for normal wave incidence on the layer was studied in (Ref. 2) for the case when a constant magnetic field was parallel to the plasma boundary.

This article investigates the behavior of the electric field of an "inflated" wave close to the "inflation" point in a magnetoactive plasma for the case of an arbitrary angle between the direction of the constant magnetic field and the normal to the plasma layer surface, and for the case of oblique wave incidence on the layer. A solution is found over a wider region for certain special cases for a wavelength which is a little less than the distance at which the layer parameters change significantly.

We shall assume that the parameters of the plasma magnetoactive layer change along the x axis, and the vector of the constant magnetic field lies in the xOz plane, forming the angle Θ with the z axis. The tensor of the dielectric constant ε_{ik} can then be written in the following form

$$\varepsilon_{ik} = \begin{pmatrix} \varepsilon_1 \cos^2\Theta + \varepsilon_3 \sin^2\Theta & i\varepsilon_2 \cos\Theta & (\varepsilon_3 - \varepsilon_1)\cos\Theta\sin\Theta \\ -i\varepsilon_2\cos\Theta & \varepsilon_1 & i\varepsilon_2\sin\Theta \\ (\varepsilon_3 - \varepsilon_1)\cos\Theta\sin\Theta & -i\varepsilon_2\sin\Theta & \varepsilon_1\sin^2\Theta + \varepsilon_3\cos^2\Theta \end{pmatrix}. \qquad (1)$$

The quantities ε_1, ε_2, ε_3 will be real, if dissipative processes are not taken into account.

Assuming that the dependence of the electromagnetic field strength on time, y and z, is determined by the factor $e^{i(-\omega t + k_y y + k_z z)}$, we obtain the following system of equations from the Maxwell equation for the electric field components /90

$$\left.\begin{aligned}
&\left(k^2 - \frac{\omega^2}{c^2}\varepsilon_{11}\right)E_x + i\left(k_y\frac{\partial E_y}{\partial x} + k_z\frac{\partial E_z}{\partial x}\right) - \frac{\omega^2}{c^2}(\varepsilon_{12}E_y + \varepsilon_{13}E_z) = 0\\
&ik_y\frac{\partial E_x}{\partial x} - \frac{\omega^2}{c^2}\varepsilon_{21}E_x - \frac{\partial^2 E_y}{\partial x^2} + \left(k_z^2 - \frac{\omega^2}{c^2}\varepsilon_{22}\right)E_y -\\
&\qquad - \left(k_yk_z + \frac{\omega^2}{c^2}\varepsilon_{23}\right)E_z = 0\\
&ik_z\frac{\partial E_x}{\partial x} - \frac{\omega^2}{c^2}\varepsilon_{31}E_x - \frac{\partial^2 E_z}{\partial x^2} - \left(k_yk_z + \frac{\omega^2}{c^2}\varepsilon_{32}\right)E_y +\\
&\qquad + \left(k_y^2 - \frac{\omega^2}{c^2}\varepsilon_{33}\right)E_z = 0
\end{aligned}\right\}, \tag{2}$$

where $k^2 = k_y^2 + k_z^2$.

The following inequality is fulfilled in the vicinity of the "inflation" point for an "inflated" field

$$\left|\frac{\partial\vec{E}}{\partial x}\right| \gg k\left|\vec{E}\right|.$$

This enables us to represent the field $\vec{E}$ in this region in the form of a series

$$\vec{E} = \vec{E}^{(0)} + \vec{E}^{(1)} + \ldots,$$

where

$$\left|\vec{E}^{(1)}\right| \sim |kx|\left|\vec{E}^{(0)}\right| \ll \left|\vec{E}^{(0)}\right|.$$

We shall always select the origin along the x axis, so that x = 0 at the inflation point. We obtain the following from the two lower equations of system (2)

$$E_y^{(0)} = E_z^{(0)} = 0,\ \frac{\partial E_y^{(1)}}{\partial x} = ik_yE_x^{(0)} + C_y,\ \frac{\partial E_z^{(1)}}{\partial x} = ik_zE_x^{(0)} + C_z;$$
$$\frac{\partial^2 E_y^{(2)}}{\partial x^2} = ik_y\frac{\partial E_x^{(1)}}{\partial x} - \frac{\omega^2}{c^2}\varepsilon_{21}E_x^{(0)},\ \frac{\partial^2 E_z^{(2)}}{\partial x^2} = ik_z\frac{\partial E_x^{(1)}}{\partial x} - \frac{\omega^2}{c^2}\varepsilon_{31}E_x^{(0)},$$

where C_y and C_z are the integration constants.

Substituting the expressions obtained in the first equation (which is differentiated with respect to x) of system (2) (we shall disregard components containing $\frac{\partial \varepsilon_{12}}{\partial x}$ and $\frac{\partial \varepsilon_{13}}{\partial x}$) and taking into account (1), we /91
can write the equation for $E_x^{(0)}$:

$$\frac{\partial}{\partial x}(\varepsilon_{11}E_x^{(0)}) + i2k_z\varepsilon_{13}E_x^{(0)} + \text{const} = 0. \tag{3}$$

It can be seen from this equation that "inflation" can only occur where ε_{11} vanishes. In the given approximation, it must be assumed that ε_{13} is constant, and is limited to the linear component in Taylor expansion for ε_{11}. Then the equation for $E_x^{(0)}$ assumes the following form

$$\frac{\partial E_x^{(0)}}{\partial x} + \left(1 + i2\frac{k_z\varepsilon_{13}}{\varepsilon'_{11}}\right)\frac{E_x^{(0)}}{x} + \frac{\text{const}}{\varepsilon'_{11}x} = 0, \tag{4}$$

where

$$\varepsilon'_{11} \doteq \left.\frac{\partial \varepsilon_{11}}{\partial x}\right|_{x=0}.$$

We can write the solution of equation (4) in the form of a sum of particular solution and the general solution of the corresponding homogeneous equation. The particular solution is a constant quantity, and is of no interest. The general solution of the homogeneous equation for the field E_x close to the "inflation" point yields the following expression

$$E_x = \frac{A}{x}e^{-i\sigma\ln(kx)}, \tag{5}$$

where A is a quantity which is constant with respect to x;

$$\sigma = 2\frac{k_z\varepsilon_{13}}{\varepsilon'_{11}}.$$

If $\sigma \neq 0$, we have

$$E_y = -k_y\frac{A}{\sigma}e^{-i\sigma\ln(kx)} + C_1;\ E_z = -k_z\frac{A}{\sigma}e^{-i\sigma\ln(kx)} + C_2,$$

where C_1 and C_2 are integration constants.

In the case of $\sigma = 0$, we have

$$E_y = ik_yA\ln(kx);\ E_z = ik_zA\ln(kx).$$

Thus, if $k_z = 0$ or $\varepsilon_{13} = 0$, the nature of the field "inflation" $\vec{E}$ is the same as in an isotropic medium.

If $k_z \neq 0$ and $\varepsilon_{13} \neq 0$, the closer the "inflation" point, the more intensely does the field $\vec{E}$ oscillate on both sides of it. Thus, the amplitude of the field component E_x increases as $\frac{1}{x}$, and the amplitudes of the components E_y and E_z remain finite. It can be seen from equation (3) that if $\frac{\partial \varepsilon_{11}}{\partial x} = 0$, at the point where $\varepsilon_{11} = 0$ "inflation" does not occur in the case of $k_z \varepsilon_{13} \neq 0$. /92

Let us investigate the case when the medium parameters change slowly and the method of geometric optics may be employed when solving the system of equations (2) outside of the vicinity of the "inflation" point

$$\vec{E} = \vec{A}(x)\, e^{i\int\limits_0^x \varkappa(x)dx}. \tag{6}$$

Substituting expression (6) in the system (2), we obtain the following equation of the fourth power for $\varkappa(x)$

$$\varepsilon_{11}\varkappa^4 + 2k_z\varepsilon_{13}\varkappa^3 + \alpha(x)\varkappa^2 + \beta(x)\varkappa + \gamma(x) = 0. \tag{7}$$

We shall not give the expressions for the coefficients α, β, γ, due to their cumbersome nature.

If k_z and ε_{13} are different from zero, we have the following from equation (7) for the wave vector component $\varkappa(x)$ of an "inflated" wave close to the "inflation" point

$$\varkappa \simeq -2k_z \frac{\varepsilon_{13}}{\varepsilon_{11}}.$$

This result coincides with formula (5). The wave oscillates in space on both sides of the "inflation" point.

1. If the magnetic field is parallel to the plasma layer surface ($\Xi = 0$), ε_{13} and $\beta(x)$ vanish and equation (7) becomes a biquadratic equation. We than have

$$\varkappa^2 = -k_y^2 - \frac{\varepsilon_1 + \varepsilon_3}{2\varepsilon_1} k_z^2 - \frac{\omega^2}{c^2} \cdot \frac{\varepsilon_2^2 - \varepsilon_1(\varepsilon_1 + \varepsilon_3)}{2\varepsilon_1} \pm$$
$$\pm \frac{1}{2\varepsilon_1}\sqrt{\left[(\varepsilon_1 - \varepsilon_3)\left(k_z^2 - \frac{\omega^2}{c^2}\varepsilon_1\right) + \frac{\omega^2}{c^2}\varepsilon_2^2\right]^2 + 4\frac{\omega^2}{c^2}k_z^2\varepsilon_2^2\varepsilon_3}. \tag{8}$$

Taking into account the components of subsequent order in the expansion with respect to the small parameter $\frac{\partial \varkappa}{\partial x}/\varkappa^2$ in the system of equations (2), we obtain the expression for the vector components $\vec{A}(x)$:

$$A_x = C\frac{\zeta^+}{\sqrt{Y}}e^{i\int\limits_0^x \frac{Z}{Y}dx};\ A_y = C\frac{\xi^-}{\sqrt{Y}}e^{i\int\limits_0^x \frac{Z}{Y}dx};\ A_z = C\frac{\eta}{\sqrt{Y}}e^{i\int\limits_0^x \frac{Z}{Y}dx}, \tag{9}$$

where C is the integration constant corresponding to one of the four functions $\varkappa(x)$; /93

$$Y = k_y(\xi^+\zeta^+ + \xi^-\zeta^-) + k_z(\zeta^+ + \zeta^-)\eta - 2\varkappa(\xi^+\xi^- + \eta^2);$$

$$\xi^{\pm} = k_z\left(k_yK^2 \pm i\frac{\omega^2}{c^2}\varepsilon_2\varkappa\right);\quad \zeta^{\pm} = k_z\left(\varkappa K^2 \pm i\frac{\omega^2}{c^2}\varepsilon_2 k_y\right);$$

$$\eta = K^2\left(k_z^2 - \frac{\omega^2}{c^2}\varepsilon_1\right) - \frac{\omega^4}{c^4}\varepsilon_2^2;\quad K^2 = \varkappa^2 + k^2 - \frac{\omega^2}{c^2}\varepsilon_1;$$

$$Z = -\frac{\omega^2}{c^2}k_yk_z^2\left[2\frac{\omega^2}{c^2}\varepsilon_2K^2\left(\frac{\partial\varepsilon_1}{\partial x} + \frac{\partial\varepsilon_2}{\partial x}\right) + \left(K^2 - \frac{\omega^2}{c^2}\varepsilon_2\right)^2\frac{\partial\varepsilon_2}{\partial x}\right].$$

If we may disregard dissipation, in the region where $\varkappa$ is real, Y and Z are real, and the exponential factors in equations (9) characterize only the wave phase.

As may be seen from (8), when ε_1 strives to zero (in the case of $\Xi = 0$ $\varepsilon_{11} = \varepsilon_1$) one value of $\varkappa^2$ goes to a finite limit and the other becomes infinite as $\frac{1}{\varepsilon_1}$:

$$\varkappa_1^2\Bigg|_{\varepsilon_1=0} = -k_y^2 + \varepsilon_3\frac{\frac{\omega^4}{c^4}\varepsilon_2^2 - k_z^4}{\frac{\omega^2}{c^2}\varepsilon_2^2 + k_z^2\varepsilon_3};\quad \varkappa_2^2\Bigg|_{\varepsilon_1\to 0} \to \frac{-1}{\varepsilon_1}\left(k_z^2\varepsilon_3 + \frac{\omega^2}{c^2}\varepsilon_2^2\right).$$

In the case of $\varkappa^2 = \varkappa^2{}_1$, the solutions of (9) are valid for the point where $\varepsilon_1 = 0$. They are not "inflated". The remaining two solutions of system (2) for the vicinity of the point where $\varepsilon_1 = 0$ cannot be represented in the form of formula (9). When trying to determine them, one may assume that ε_2 and ε_3 are constant, and for ε_1 one may restrict oneself to the linear components in the Taylor expansion:

$$\varepsilon_1 \simeq \varepsilon_1' x.$$

After several cumbersome computations, we obtained the following equation from the system of equations (2) for the field component E_x

$$\varepsilon_1' x \frac{\partial^4 E_x}{\partial x^4} + 4\varepsilon_1' \frac{\partial^3 E_x}{\partial x^3} - \left(\frac{\omega^2}{c^2}\varepsilon_2^2 + k_z^2\varepsilon_3\right)\frac{\partial^2 E_x}{\partial x^2} + \left(\frac{\omega^2}{c^2}\varepsilon_3 - k^2\right)\left(\varepsilon_1' x \frac{\partial^2 E_x}{\partial x^2} + \right.$$
$$\left. + 2\varepsilon_1' \frac{\partial E_x}{\partial x}\right) - \left[\frac{\omega^2}{c^2}\varepsilon_2^2\left(\frac{\omega^2}{c^2}\varepsilon_3 - k_y^2\right) - k^2 k_z^2 \varepsilon_3 + \frac{\omega^2}{c^2} k_z^2 \varepsilon_1 \varepsilon_3\right] E_x = 0.$$

Disregarding the last two components, we may write

$$\frac{d^2 u}{dx^2} + \frac{4}{x} \cdot \frac{du}{dx} - \frac{1}{x} pu = 0, \tag{10}$$

where /94

$$u = \frac{\partial^2 E_x}{\partial x^2}; \quad p = \frac{\frac{\omega^2}{c^2}\varepsilon_2^2 + k_z^2 \varepsilon_3}{\varepsilon_1'}.$$

If we substitute $u(x) = \xi^{-3} v(\xi)$, $x = \xi^2$, equation (10) becomes a Bessel equation

$$\frac{d^2 v}{d\xi^2} + \frac{1}{\xi} \cdot \frac{dv}{d\xi} - \left(4p + \frac{9}{\xi^2}\right) v = 0,$$

and, employing the recurrent formulas for the Bessel function, we obtain

$$E_x = \frac{A I_1 (2\sqrt{px}) + B K_1 (2\sqrt{px})}{\sqrt{px}}, \tag{11}$$

where A and B are the integration constants; $I_1(2\sqrt{px})$ and $K_1(2\sqrt{px})$ -- the McDonald and Bessel functions. In the region under consideration, the components of the field $\vec{E}$ are related by the following relationships

$$\frac{\partial^2 E_y}{\partial x^2} \simeq i k_y \frac{\partial E_x}{\partial x}; \quad \frac{\partial^2 E_z}{\partial x^2} \simeq i k_z \frac{\partial E_x}{\partial x},$$

which enable us to determine E_y and E_z:

$$\frac{E_y}{k_y} = \frac{E_z}{k_z} = \frac{i}{p}\{A I_0 (2\sqrt{px}) - B K_0 (2\sqrt{px})\}. \tag{12}$$

Taking the asymptotic form of the Bessel function and requiring that expressions (11) and (12) change into expression (6) for large values of $|x|$, we may find the relationship between the coefficients A and B and the coefficients C included in (9). In the case of $p > 0$, the coefficients C are related to A and B in the following way to the right of the "inflation" point (× is imaginary in this case):

$$C_+ = -\frac{1}{(-i)^{1/2}}\left(\frac{\omega^2\varepsilon_1'}{2\pi c^2}\right)^{1/2}\frac{A}{p}, \quad C_- = \frac{1}{i^{1/2}}\left(\frac{\omega^2\varepsilon_1'}{2\pi c^2}\right)^{1/2}\frac{iA - \pi B}{p},$$

where $C = C_+$ in the case of $\varkappa$ imaginary negative; $C = C_-$ in the case of $\varkappa$ imaginary positive.

2. If the magnetic field is perpendicular to the layer surface ($\Xi = \frac{\pi}{2}$), the system of equations (2) may be reduced to a system of two second order equations /95

$$\left.\begin{aligned} &\frac{d^2\varphi}{dx^2} + \left(\frac{\omega^2}{c^2}\varepsilon_1 - k^2\right)\varphi + i\frac{\omega^2}{c^2}\varepsilon_2\psi = 0 \\ &\varepsilon_3\frac{d^2\psi}{dx^2} - \frac{k^2\frac{\partial\varepsilon_3}{\partial x}}{\frac{\omega^2}{c^2}\varepsilon_3 - k^2}\cdot\frac{d\psi}{dx} + \left(\frac{\omega^2}{c^2}\varepsilon_3 - k^2\right)(\varepsilon_1\psi - i\varepsilon_2\varphi) = 0 \end{aligned}\right\}, \tag{13}$$

where
$$E_y = \frac{1}{k^2}(k_y\psi + k_z\varphi); \quad E_z = \frac{1}{k^2}(k_z\psi - k_y\varphi); \quad E_x = \frac{i}{\frac{\omega^2}{c^2}\varepsilon_3 - k^2}\cdot\frac{d\psi}{dx}.$$

Outside of the vicinity of the "inflation" point, and representing ϕ and ψ similarly to (6) in the following form, we have:

$$\varphi, \psi = A_{\varphi,\psi}(x)e^{i\int_0^x \varkappa dx},$$

We obtain the following from the system of equations (13) in the approximation of geometric optics

$$\varkappa^2 = \frac{\omega^2}{c^2}\varepsilon_1 - \frac{k^2}{2}\left(1 + \frac{\varepsilon_1}{\varepsilon_3}\right) \pm \frac{1}{2\varepsilon_3}\sqrt{k^4(\varepsilon_1 - \varepsilon_3)^2 + 4\frac{\omega^2}{c^2}\varepsilon_2^2\varepsilon_3\left(\frac{\omega^2}{c^2}\varepsilon_3 - k^2\right)};$$
$$A_\varphi = i\frac{\omega^2}{c^2}\varepsilon_2 C Q^{1/2}; \quad A_\psi = K^2 C Q^{1/2}, \tag{14}$$

where
$$Q = \frac{\frac{\omega^2}{c^2}\varepsilon_3 - k^2}{\varkappa\left[\left(K^4 + \frac{\omega^4}{c^4}\varepsilon_2^2\right)\varepsilon_3 - \frac{\omega^2}{c^2}k^2\varepsilon_2^2\right]}; \quad K^2 = \varkappa^2 + k^2 - \frac{\omega^2}{c^2}\varepsilon_1;$$

C is an integration constant corresponding to one of the four functions $\varkappa(x)$.

As may be seen from (14), close to the point $\varepsilon_3 = 0$ (in the case of $\Xi = \frac{\pi}{2}\,\varepsilon_{11} = \varepsilon_3$) one solution for $\varkappa^2$ is finite, and the other solution has the following form

$$\varkappa^2 \approx -k^2 \frac{\varepsilon_1}{\varepsilon_3}.$$

In the first case, solutions of (14) are valid at the point $\varepsilon_3 = 0$. They correspond to a "non-inflated" wave. In the solution of system (13), /96
we shall assume that ε_1 and ε_2 are constant, and we shall expand ε_3 in Taylor series in the vicinity of the point $\varepsilon_3 = 0$, and shall confine ourselves to the linear component

$$\varepsilon_3 \simeq \varepsilon_3' x.$$

We may find the following from the first equation of system (13)

$$\psi = i\frac{c^2}{\omega^2\varepsilon_2}\left[\frac{d^2\varphi}{dx^2} + \left(\frac{\omega^2}{c^2}\varepsilon_1 - k^2\right)\varphi\right] \approx i\frac{c^2}{\omega^2\varepsilon_2}\cdot\frac{d^2\varphi}{dx^2}. \tag{15}$$

Substituting this expression in the second equation of system (13), disregarding the small components, and making the substitution $v = \frac{d^2\phi}{dx^2}$, we obtain

$$\frac{d^2v}{dx^2} + \frac{1}{x}\cdot\frac{dv}{dx} + p\frac{v}{x} = 0,$$

where

$$p = -\frac{k^2\varepsilon_1}{\varepsilon_3'}.$$

When $\xi = 2\sqrt{px}$ is used as the independent variable, this equation is reduced to a Bessel equation

$$\frac{d^2v}{d\xi^2} + \frac{1}{\xi}\cdot\frac{dv}{d\xi} + v = 0.$$

Utilizing (15), we find the following from the latter equation

$$\psi = C_1 H_0^{(1)}(2\sqrt{px}) + C_2 H_0^{(2)}(2\sqrt{px}), \tag{16}$$

where C_1, C_2 are the integration constants; $H_0^{(1)}(\xi)$, $H_0^{(2)}(\xi)$ -- Hankel functions.

As may be seen from (15), close to the "inflation" point $\phi|\sim|\varepsilon_3\psi| <<$ $<< |\psi|$ -- i.e., the contribution from ϕ to the expression for the fields in this region is negligibly small. If we make the stipulation that (16) change into an expression like (14) (outside of the vicinity of the "inflation" point), we may find the relationship between the constants C_2, C_2 in (16) and the constants C in formula (14).

Just as in the case of an isotropic layer, the presence of "inflation" points in a magnetoactive, nonuniform, transmittant medium leads to the fact that the thermal radiation intensity of this medium is on the order /97 of the radiation intensity of an absolute black body, if the wavelength being studied is comparable to the distance from the "inflation" point to the layer boundary. When an electromagnetic wave, whose length is comparable to the plasma layer thickness, falls on this layer, the amount of energy absorbed by the plasma in the vicinity of the "inflation" point per unit of time (per 1 cm^2 of layer surface) is

$$S \sim \frac{c}{4\pi} E^2, \qquad (17)$$

where E is the electric field amplitude in a vacuum.

However, (17) is valid until the phenomena caused by the oscillation nonlinearity and spatial dispersion in the vicinity of the "inflation" point exceed the phenomena caused by particle collisions in this region, i.e.,

$$|(\vec{v}\nabla)\vec{v}| \lessapprox |\nu\vec{v}|; \; K\sqrt{\frac{T}{m}} \lessapprox \nu,$$

where $\vec{v}$ is the electron velocity caused by the wave field; ν -- the effective frequency of electron collisions with ions; $K \sim \frac{\omega}{\nu} \cdot \frac{1}{\ell}$ -- effective wave vector in the vicinity of the "inflation point; ℓ -- layer thickness; m -- electron mass; T - temperature. This leads to a limitation for the field E and the temperature T, which can be fulfilled by (17):

$$E \lessapprox \frac{\nu^3 l}{\omega^2 c} H_0; \; T \lessapprox \left(\frac{\pi L^2 e^8 n_0^2 l}{m^{1/2}\omega}\right)^{2/3},$$

where H_0 -- the constant magnetic field strength; L -- Coulomb logarithm; n_0 -- electron density in the plasma; e -- electron charge. The maximum energy obtained by a charged particle per unit of time is

$$w \sim \frac{\nu^6 l H_0^2}{10\omega^4 c n_0}.$$

In the case of $n_0 = 10^{15}$ cm^{-3}, $T = 10^6$°K, $H_0 = 10^5$G, $\omega \sim 5 \cdot 10^{10}$ sec^{-1}, $w \sim 100$ ev/sec, $E \lesssim 1$ v/cm.

Thus, it is not advantageous to employ "inflated" fields for plasma heating.

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EXCITATION OF A MAGNETOHYDRODYNAMIC WAVE GUIDE IN A COAXIAL LINE

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As is known (Ref. 1), magnetohydrodynamic waves have low frequencies ($\omega \ll \Omega_H \lesssim 1.5 \cdot 10^8 \frac{1}{\text{sec}}$, with respect to $\lambda_{vac} \gg 10^2$ cm). At these frequencies, a coaxial line represents the most reasonable method for transmitting energy from an oscillator to a wave guide. Therefore, the problem of excitation of an axially symmetrical E-wave in a magnetohydrodynamic wave guide by a TEM wave of a coaxial line is of great interest. If the plasma temperature may be disregarded, and its conductivity is great enough, under these conditions, the plasma may be characterized by the dielectric constant tensor without spatial dispersion. The problem under consideration is then a special case of the problem regarding the matching between an anisotropic dielectric wave guide and a coaxial line. Equations for determining the stray field were obtained in (Ref. 2, 3):

$$\varphi(t) + \frac{1}{2(\pi i)^2} \int_{-\infty}^{+\infty} \frac{Z(t') Z_0(t')\, dt'}{[Z(t') + Z_0(t')](t' - t)} \int_{-\infty}^{+\infty} \left[\frac{1}{Z_0(t'')} - \frac{1}{Z_0(t')}\right] \frac{\varphi(t'')\, dt''}{t'' - t'} = \tag{1}$$

$$= -\frac{k}{(\pi i)^2 a} \int_{-\infty}^{+\infty} \frac{Z_0(t') Z(t')\, dt'}{[Z(t') + Z_0(t')](t'^2 - k^2)(t' - t)}; \quad \mathrm{Im}\, k > 0.$$

The diffracted magnetic field in the space between the wave guide (radius a) and the casing (radius b) is expressed by means of the indeterminate function $\phi(t) \equiv \phi^{+}(t) - \phi^{-}(t) \equiv \phi^{+}(t) - \phi^{+}(-t)$ (the indices $(\pm)$ designate the analyticity in the upper or the lower half plane of the complex variable t): /99

$$H(z)=\int_{-\infty}^{+\infty}\frac{k}{v}\cdot\frac{\Delta_1(r)}{\Delta_0(a)}\varphi^{-}(t)\,e^{itz}dt \text{ при } z<0; \tag{2}$$

$$H(z)=\int_{-\infty}^{+\infty}\frac{k}{v}\cdot\frac{\Delta_1(r)}{\Delta_0(a)}\cdot\frac{1}{Z_0^{-1}(t)+Z^{-1}(t)}\left\{\varkappa^{+}(t)-\frac{1}{Z}\varphi^{+}(t)-\frac{1}{2\pi i}\times\right.$$

$$\left.\times\frac{1}{t-k}\right\}\exp(itz)\,dt \text{ for } z>0, \tag{3}$$

where

$$\varkappa^{+}(t)-\varkappa^{+}(-t)=-\frac{1}{Z_0(t)}\varphi(t)+\frac{t}{\pi i a(t^2-k^2)};$$

$$Z_0(t)=\frac{v}{k}\cdot\frac{\Delta_0(a)}{\Delta_1(a)};\quad Z(t)=\frac{\beta(t)}{k\varepsilon_{11}}\cdot\frac{I_0(\beta a)}{I_1(\beta a)};$$

$$\Delta_n(r)=I_n(vr)\,K_0(vb)-(-1)^n K_1(vr)\,I_0(vb),\ n=0,1;$$

$$v=(t^2-k^2)^{1/2};\quad \beta=\left[\frac{\varepsilon_{\parallel}}{\varepsilon_{\perp}}(t^2-k^2\varepsilon_{\perp})\right]^{1/2}.$$

If the plasma density is large ($\omega_0^2 a^2 >> c^2$), equation (1) can be solved according to the iteration method.

The reflection coefficient of a coaxial wave in the first approximation, as a function of the parameter $\frac{\delta}{a\ \ln(b/a)}$, is

$$R=-\frac{\delta}{2a\ln(b/a)}\left[1-\frac{2}{3}(\varepsilon_{\perp}-1)^{1/2}\right],\ \varepsilon_{\perp}-1\ll 1;$$

$$R=-\frac{\delta}{3\varepsilon_{\perp}a\ln(b/a)},\ \varepsilon_{\perp}\gg 1. \tag{4}$$

When the amplitude of three-dimensional waves excited in a magnetohydrodynamic wave guide is computed, a distinction must be drawn between the four regions of the plasma parameters and the wave parameters. The magnetic field amplitudes have the following values corresponding to these regions (the amplitude of the coaxial wave equals unity):

$$T_n=\frac{2}{\ln(b/a)}[\lambda_n^2+\ln^{-2}(b/a)]^{-1},\qquad \varepsilon_{\perp}-1\ll\frac{\lambda_n^2\delta^2}{a^2}\ll 1;$$

$$T_n = \frac{2}{\lambda_n^2 \ln (b/a)} \left[\frac{\lambda_n^2 \delta^2}{a^2 (\varepsilon_\perp - 1)}\right]^2, \qquad \frac{\delta^2 \lambda_n^2}{a^2} \ll \varepsilon_\perp - 1 \ll 1;$$

$$T_n = \frac{2}{\lambda_n^2 \varepsilon_\perp \ln (b/a)} \left[\frac{\lambda_n^2 \delta^2}{a^2}\right]^2, \qquad \frac{\lambda_n^2 \delta^2}{a^2} \ll 1 \ll \varepsilon_\perp; \tag{5}$$ /100

$$T_n = \frac{2}{\lambda_n^2 \varepsilon_\perp \ln (b/a)}, \qquad \frac{\lambda_n^2 \delta^2}{a^2} \gg 1.$$

Thus, the effective excitation of magnetohydrodynamic waves by a coaxial line is only observed for strong magnetic fields ($\varepsilon_\perp - 1 \ll 1$).

Harmonics with large numbers corresponding to the limiting case $\lambda_n^2 \gg a^2/\delta^2$ are always only slightly excited. For weak magnetic fields, the main portion of the coaxial wave power is dispersed into excitation of the surface plasma wave.

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THEORY OF MAGNETOHYDRODYNAMIC WAVE SCATTERING AT THE END OF A WAVE GUIDE

V. I. Kurilko

The study of magnetohydrodynamic waves is of great interest in solving several problems of plasma physics, such as controlled thermonuclear synthesis, magnetohydrodynamic oscillators, etc. (Ref. 1). A great many articles have been recently published which investigated the propagation of magnetohydrodynamic waves in uniform, unconfined magnetohydrodynamic wave guides. However, practically every wave guide is confined. Therefore, it becomes necessary to investigate the phenomena related to the scattering of magnetohydrodynamic waves at the end of a wave guide (for example, reflection of one of the eigen waves of such a wave guide, and its transformation into other waves). In addition, the theory of electromagnetic wave scattering includes the excitation of a confined wave guide. /101

This article investigates these phenomena for a semi-infinite plasma wave guide. In the general case, this problem may be reduced to a system of two coupled integral equations or (in the presence of an infinite casing) to two infinite, coupled systems of algebraic equations, whose solution may only be found by numerical methods. Therefore, let us investigate the case when the end of the wave guide is covered by a conducting diaphragm. As will be shown below, the problem of determining the Fourier component of the scattered field may be reduced to an integral Fredholm equation of the second type, whose solution may be obtained by a numerical method, and when there is a small parameter -- in analytical form.

We shall assume that the plasma conductivity is infinite, and its temperature equals zero. In this case, the electrodynamic properties of the plasma wave guide, as is known, may be characterized by the dielectric constants $\varepsilon_{\perp} = 1 + \frac{\omega_0^2}{\omega_H \Omega_H}$, $\varepsilon_{\parallel} = 1 - \frac{\omega_0^2}{\omega^2}$ $\left(\omega_0^2 = \frac{4\pi e^2 n}{m},\ \omega_H = \frac{eH_0}{mc},\ \Omega_H = \frac{eH_0}{Mc} \gg \omega\right)$, which only depend on frequency. Since the frequency is fixed in our problem, the plasma wave guide may be regarded as a special case of an anisotropic dielectric wave guide. Therefore, let us first investigate the more general problem of electromagnetic wave scattering by the jump in the dielectric constants of an anisotropic dielectric wave guide, whose uniform sections are separated by the conducting diaphragm. The plasma wave guide parameters may be employed to find the analytical solutions of the general equation obtained.

Thus, let us investigate an anisotropic dielectric wave guide ($r < a$, $-\infty < z < +\infty$) with a piecewise-uniform tensor dielectric constant

$$\hat{\varepsilon}(z > 0,\ 0 < r < a) = \left(\varepsilon_{\parallel}^{(1)},\ \varepsilon_{\perp}^{(1)}\right);$$

$$\hat{\varepsilon}(z < 0,\ 0 < r < a) = \left(\varepsilon_{\parallel}^{(2)},\ \varepsilon_{\perp}^{(2)}\right);$$

$$\hat{\varepsilon}(-\infty < z < +\infty,\ a < r < b) \equiv 1.$$

Let us assume that the uniform sections of the wave guide are separated by an ideally conducting diaphragm (z = 0, 0 < r < a), and that the conducting casing is not confined and is located at the distance r = b from the wave guide axis. Let us assume that one of the eigen, axially symmetrical E-waves of this wave guide, which is characterized by the wave number h_m, falls on the nonuniform section from the right wave guide. Let us determine the amplitudes for the eigen waves of both wave guides which are excited due to scattering of this wave. We may write the solution for the fields in the following form /102

$$\left.\begin{aligned} H_\varphi^e &= \int\limits_{-\infty}^{+\infty} H(t)\Delta_1(t)e^{itz}dt + \exp(-ih_m z) \\ E_z^e &= \int\limits_{-\infty}^{+\infty} \frac{iv}{k} H_\perp(t)\Delta_0(t)e^{itz}dt + Z_m^1 \exp(-ih_m z) \end{aligned}\right\} \begin{aligned} &r = a+0, \\ &-\infty < z < \\ &< +\infty; \end{aligned} \tag{1}$$

$$\Delta_n(t) = I_n(va)K_0(vb) - (-1)^n K_n(va)I_0(vb), \quad n = 0,1;$$

$$v(t) = (t^2 - k^2)^{1/2}; \quad k = \frac{\omega}{c}, \quad \operatorname{Im}\omega > 0;$$

$$\left.\begin{aligned} H_\varphi^s &= \int\limits_{-\infty}^{+\infty} h_s(t)J_1(\beta_s a)e^{itz}dt + 2A_s\cos h_m z \\ E_z^s &= \int\limits_{-\infty}^{+\infty} Z^s(t)h_s(t)J_1(\beta_s a)e^{itz}dt + 2A_s Z_m^1\cos h_m z \\ E_r^s &= \int\limits_{-\infty}^{+\infty} \frac{it}{k\varepsilon_\perp^{(s)}} h_s(t)J_1(\beta_s a)e^{itz} + 2A_s h_m(\sin h_m z)\frac{1}{k\varepsilon_\perp^{(1)}} \end{aligned}\right\} \begin{aligned} &r = a-0, \\ &s = 1 - z > 0, \\ &s = 2 - z < 0, \end{aligned} \tag{2}$$

where

$$Z_s(t) = \frac{i\beta_s(t)}{k\varepsilon_\parallel^{(s)}} \cdot \frac{J_0(\beta_s a)}{J_1(\beta_s a)}; \quad \beta_s = \left[\frac{\varepsilon_\parallel^{(s)}}{\varepsilon_\perp^{(s)}}\left(\varepsilon_\perp^{(s)}k^2 - t^2\right)\right]^{1/2};$$

$$A_1 = 1; \quad A_2 = 0.$$

The fields thus selected satisfy the Maxwell equation. We can determine the remaining, unknown Fourier amplitudes H, h_s of the desired fields from the boundary conditions on the lateral surfaces of the wave guide /103 and on the conducting diaphragm. The boundary conditions on the wave guide -- vacuum surface have the following form

$$H_\varphi^{(1)} = H_\varphi^e,\ E_z^{(1)} = E_z^e \quad \text{for } r = a,\ z > 0;$$
$$H_\varphi^{(2)} = H_\varphi^e,\ E_z^{(2)} = E_z^e \quad \text{for } r = a,\ z < 0. \tag{3}$$

Substituting (1) and (2) in the boundary conditions (3) and employing the results derived in (Ref. 2), we may express the unknown functions H and h_s by the boundary values on the contour Im t = 0 of the functions which are analytical in the upper (+) and the lower (-) half-planes of the complex variable t:

$$h_1(t) J_1(\beta_1 a) = \frac{1}{D_1(t)} \left\{ Z(t)\varphi^+(t) - \psi^+(t) - \frac{Z_m^1 - Z(t)}{2\pi i (t - h_m)} \right\};$$
$$h_2(t) J_1(\beta_2 a) = \frac{1}{D_2(t)} \left\{ Z(t)\varkappa^-(t) - \xi^-(t) - \frac{Z_m^1 - Z(t)}{2\pi i (t + h_m)} \right\}; \tag{4}$$
$$H(t)\Delta_1(t) = \frac{1}{D_1(t)} \left\{ Z^1(t)\varphi^+(t) - \psi^+(t) - \frac{Z^1(t) - Z_m^1}{2\pi i (t - h_m)} \right\} \equiv$$
$$\equiv \frac{1}{D_2(t)} \left\{ Z^2(t)\varkappa^-(t) - \xi^-(t) - \frac{Z^2(t) - Z_m^1}{2\pi i (t + h_m)} \right\},$$

where

$$D_s(t) \equiv Z^s(t) - Z(t),\ Z(t) = \frac{iv}{k} \cdot \frac{\Delta_1(t)}{\Delta_0(t)}.$$

The latter equation (4) represents a boundary problem for determining the unknown functions ϕ^+, ψ^+, ξ^- and $\varkappa^-$. The relationships lacking between these functions may be determined from the boundary conditions on the conducting diaphragm

$$E_r^s(z = 0;\ 0 < r < a) = 0. \tag{5}$$

Substituting E_r^S in the boundary conditions (5), we obtain

$$h^s(-t) \equiv h^s(t). \tag{6}$$

We obtain the following by means of the latter equation and the Sommerfeld condition for the finiteness of the magnetic field and integrability of the electric field close to the diaphragm edge:

$$\varkappa_1^-(t) \equiv -\varphi_1^+(-t);\ \xi_1^-(t) \equiv -\psi_1^+(-t), \tag{7}$$

where /104

$$\varkappa_1^-(t) = \varkappa^-(t) + \frac{1}{2\pi i} \cdot \frac{1}{t - h_m}; \quad \xi_1^-(t) = \xi^-(t) + \frac{Z_m^1}{2\pi i\,(t - h_m)};$$

$$\psi_1^+(t) = \psi^+(t) + \frac{Z_m^1}{2\pi i\,(t + h_m)}; \quad \varphi_1^+(t) = \varphi^+(t) + \frac{1}{2\pi i} \cdot \frac{1}{t + h_m},$$

$$\operatorname{Im} h_m > 0.$$

(Equations (6) are only necessary for fulfilling the boundary conditions (5). We shall assume that these equations are sufficient, although this may only be proven for a rectangular wedge [Ref. 3]).

Thus, the boundary problem for determining the function assumes the following form

$$\frac{1}{D_1(t)}\left\{Z^1(t)\,\varphi_1^+ - \psi_1^+(t) + \frac{2h_m}{\pi i} \cdot \frac{Z^1(t) - Z_m^1}{t^2 - h_m^2}\right\} \equiv$$
$$\equiv \frac{1}{D_2(t)}\left\{- Z^2(t)\,\varphi_1^-(t) + \psi_1^-(t) - \frac{2h_m}{\pi i} \cdot \frac{Z^2(t) - Z_m^1}{t^2 - h_m^2}\right\}. \tag{8}$$

This type of boundary problem for one special case ($\hat{\varepsilon}_1 \equiv 1$, $\hat{\varepsilon}_2 \to \infty$) was first studied in (Ref. 4). It was shown that in the presence of a casing, when the coefficients have singularities of only the pole type, it can be reduced to an infinite system of algebraic equations. However, by employing the formulas of Sokhotskiy -- Plemel', it is more advantageous to reduce the problem (8) to an integral equation. By combining and subtracting equation (8) with its mirror image at the point t = 0, we obtain (Ref. 5)

$$\frac{1}{\pi i}\int_{-\infty}^{+\infty} \frac{\psi_1(t')\,at'}{t' - t} + \frac{1}{2\pi i} Z(t) \sum_{s=1}^{2} \frac{Z^s(t)}{D_s(t)} \int_{-\infty}^{+\infty} \left[\frac{1}{Z(t')} - \frac{1}{Z(t)}\right] \frac{\psi_1(t')}{t' - t}\,dt' =$$
$$= -\frac{h_m}{\pi i} \cdot \frac{Z(t)}{t^2 - h_m^2} \sum_{s=1}^{2} \frac{Z^s(t) - Z_m^1}{D_s(t)} \equiv f(t); \quad Z(t)\,\varphi_1(t) = \psi_1(t). \tag{9}$$

The index of the latter equation equals zero (Ref. 5). Therefore, it is /105
is equivalent to one integral Fredholm equation of the second type for $\psi_1(t) = \psi_1^+(t) - \psi_1^+(-t)$:

$$\psi_1(t) + \frac{1}{2(\pi i)^2}\int_{-\infty}^{+\infty}\frac{Z(t')\,dt'}{t'-t}\sum_{s=1}^{2}\frac{Z^s(t')}{D_s(t')}\int_{-\infty}^{+\infty}\left[\frac{1}{Z(t'')}-\frac{1}{Z(t')}\right]\frac{\psi_1(t'')\,dt''}{t''-t'} = \tag{10}$$

$$= \frac{1}{\pi i}\int_{-\infty}^{+\infty}\frac{dt'}{t'-t}f(t').$$

According to the general theory of singular equations with a kernel of the Cauchy type (Ref. 5), the solution of (9) which vanishes in the case of $t \to \infty$ exists and is real. Due to the equivalence between (9) and (10), the same holds for the solution of equation (10). It can be solved numerically in the general case, and in the presence of parameters -- it can be solved analytically, even if there is no casing ($b \to \infty$) and the coefficients in equation (8) have singularities of the point branching type.

In the case of the magnetohydrodynamic wave guide, which we are investigating, the ratio between the wave guide radius and the wave length in a vacuum serves as one of the large parameters. Even for $H_0 \sim 10^4$ gauss, $\lambda_0 \equiv \frac{c}{\Omega_H} \sim 10^2$ cm. Therefore, in the case of $a \lesssim 5$ cm the ratio a/λ (λ -- wavelength in a vacuum) cannot exceed 10^{-2} (with allowance for the requirement that $\lambda >> \lambda_0$), so that even In (λ/a) is large (In $(\lambda/a) \gtrsim 4$). In addition, in a significant number of important cases the linear plasma density na^2 is great, so that the ratio $\frac{\delta}{a} = \frac{c}{\omega_0 a}$ is small.

Assuming that the inequality $\delta << a << \lambda$ is fulfilled, we can significantly simplify equation (10). Disregarding terms on the order of /106 $(\delta/a)\text{In}(\lambda/a)$, $\left(\frac{a}{\lambda}\right)^2\text{In}(\lambda/a)$ and taking the fact into account that $\hat{\varepsilon}_2 \equiv 1$, we obtain

$$\frac{1}{\pi i}\int_{-\infty}^{+\infty}\frac{\psi(t')\,dt'}{t'-t} + \frac{Z(t)}{\pi i}\int_{-\infty}^{+\infty}\frac{\psi(t')\,dt'}{Z(t')(t'-t)} = -\frac{2h_m}{\pi i}\cdot\frac{Z(t)}{t^2-h_m^2}\left[1+\frac{Z_m^1}{Z(t)}\right] \tag{10'}$$

(the index « 1 » for ψ is omitted from this point on). Equation (10'), as may be readily seen, is equivalent to the scalar boundary problem

$$\psi^+(t) = Z(t)\left\{u^-(t) - \frac{h_m}{\pi i}\cdot\frac{1}{t^2-h_m^2}\left[1+\frac{Z_m^1}{Z(t)}\right]\right\}. \tag{11}$$

The solution of (11) has the following form

$$\psi^{+}(t)=\frac{(t+k)^{1/2}}{2\pi i}X^{+}(t)\int\limits_{C}\frac{dt'}{t'-t}\cdot\frac{ih_m Z(t')}{\pi\left(t'^2-h_m^2\right)}\left[1+\frac{Z_m^1}{Z(t')}\right]\frac{(t'-k)^{1/2}}{X^{+}(t')}, \tag{12}$$

where

$$X^{+}(t)=\frac{K_0(va)}{K_1(va)}X^{-}(t);\quad X^{+}(t)=\frac{1}{X^{-}(-t)},$$

and the singularity at the point $t' = t$ passes around the contour C from below.

Within an accuracy of a term on the order of $\left(\frac{a}{\lambda}\right)^2 \ln\frac{\lambda}{a} \ll 1$, we have

$$X^{+}(t)=(ta+ka)^{1/2}\exp\left\{\frac{ta}{\pi}\int\limits_{ka}^{\infty}\frac{xdx}{\sqrt{x^2-k^2a^2}}\cdot\frac{1}{x^2+t^2a^2}\times\right. \tag{12'}$$
$$\left.\times\ln\left[\ln\left(\frac{\pi}{2}-i\ln\frac{2}{\gamma x}\right)\right]\right\},$$

where γ is the Euler constant, and in the case of $\ln\frac{\lambda}{a} \gg 1$ we have

$$X^{+}(t)=\left[\frac{\pi}{2}-i\ln\frac{2\lambda}{\gamma a}\right]^{1/2}(ta+ka)^{1/2}\left\{1-\frac{1}{4}\cdot\frac{\ln(1+t/k)}{\ln\lambda/a}\right\}.$$

Expansion in powers of $\left|\ln\frac{\lambda}{a}\right|^{-1}$ is usually employed in antenna theory (Ref. 6), and numerical integration may be employed to determine expression (12') for $\frac{a}{\lambda}X+(t)$ which are not too small.

In the same approximation ($\delta \ll a \ll \Lambda$), the boundary problem for determining the function $\phi^{+}(t)$ has the following form

$$\varphi^{+}(t)=\frac{1}{Z(t)}\omega^{-}(t)-\frac{h_m}{\pi i}\cdot\frac{1}{t^2-h_m^2}\left[1+\frac{Z_m^1}{Z(t)}\right]. \tag{13}$$

The integral in the solution of (12) may be readily computed by closing the integration contour in the lower halfplane in the first term, and in the upper halfplane in the second term, by the complex variable t. The integral in the solution of (13) may be computed similarly for $\phi^{+}(t)$. Substituting the expressions thus obtained for $\psi^{+}(t)$ and $\phi^{+}(t)$ into /107 the formula for the amplitude of the Fourier-field in a vacuum

$$\Delta_1(t)H(t)=\frac{1}{D_1(t)}\left\{Z^1(t)\varphi^+(t)-\psi^+(t)+\frac{h_m}{\pi i}\cdot\frac{Z^1(t)-Z^1_m}{t^2-h^2_m}\right\}, \tag{14}$$

by means of (1) we may find the expression for the amplitude for a wave with the number 1, which is excited in a wave guide (z > 0) during the scattering of a wave with the number m:

$$\begin{aligned} R_{l_m}&=\frac{2\pi i}{D'_{1l}}\left\{Z^1_l\varphi^+_l-\psi^+_l+\frac{h_m}{\pi i}\cdot\frac{Z^1_l-Z^1_m}{h^2_l-h^2_m}\right\}\equiv\\ &\equiv\frac{k^{-1}}{D'_{1l}}\left\{X^+_mX^+_l\frac{[(n_m+1)(n_l+1)]^{1/2}}{n_m+n_l}-2Z^1_l n_m\left[\frac{1}{n^2_l-n^2_m}+\right.\right.\\ &+\left.\frac{1}{2n_m(n_m-n_l)}\left(\frac{n_m+1}{n_l+1}\right)^{1/2}\frac{X^+_m}{X^+_l}\right]+2Z^1_m n_m\left[\frac{1}{n^2_l-n^2_m}+\right.\\ &+\left.\frac{1}{2n_l(n_m-n_l)}\left(\frac{n_l+1}{n_m+1}\right)^{1/2}\frac{X^+_l}{X^+_m}\right]+2n_m\frac{Z^1_l-Z^1_m}{n^2_l-n^2_m}-\\ &-\left.\frac{Z^1_lZ^1_m}{(n_m+n_l)[(n_m+1)(n_l+1)]^{1/2}X^+_lX^+_m}\right\},\quad n_{l,m}\equiv\frac{h_{l,m}}{k}, \end{aligned} \tag{15}$$

where $D'_{1l}\equiv\frac{d}{dt}D_1|_{t=h_l}$; $Z^1_{lm}\equiv Z^1(h_{l,m})$; $X^+_{l,m}\equiv X^+(h_{l,m})$;

$$\varphi^+_l=\varphi^+(h_l);\ \psi^+_l=\psi^+(h_l).$$

The latter expression may be significantly simplified in the most interesting cases of small and large retardations $n_{\ell,m}$. It thus appears that the sum in the parentheses in (15) is represented in the form of a power series of the logarithms for the ratios λ/a and λ/δ. Retaining the old terms in this series, we obtain

$$\begin{gathered} R_{l_m}=-\frac{2\ln\frac{\lambda^2}{a\delta}}{\gamma^2_l\ln^2\frac{\lambda}{\delta\nu_l}+2\ln\frac{\lambda}{\delta\nu_l}};\quad n_\perp-1\ll\frac{\delta^2\nu^2_{l,m}}{a^2}\ll1;\\ J_1(\nu_{l,m})=0;\ n^2_\perp=\varepsilon_\perp;\\ R_{l_m}=\left[\frac{\delta^2\nu_l}{a^2(\varepsilon_\perp-1)}\right]^2\frac{\ln(\varepsilon_\perp-1)}{\ln^2\frac{\lambda}{a(\varepsilon_\perp-1)^{1/2}}}; \end{gathered} \tag{16}$$

$$\frac{\delta^2\nu^2_{l,m}}{a^2} \ll n_\perp - 1 \ll 1; \qquad (17)$$ /108

$$R_{l_m} = \frac{i\delta^4\nu_l^2}{a^4\varepsilon_\perp} \cdot \frac{\ln^2\frac{\lambda}{n_\perp a} + \ln\frac{\lambda}{a}\ln\frac{\lambda}{an_\perp^{3/2}}}{\ln\frac{\lambda}{a}\ln^2\frac{\lambda}{an_\perp}};$$

$$\frac{\delta^2\nu^2_{l,m}}{a^2} \ll 1 \ll n_\perp \ll \frac{\lambda}{a}; \qquad (18)$$

$$R_{l_m} = \frac{\nu_m}{\nu_l^2(\nu_l + \nu_m)\varepsilon_\perp} \cdot \frac{\ln\frac{\lambda a}{\delta^2\nu_l\nu_m}\ln\frac{\lambda}{a} + \ln\frac{\lambda}{\delta\nu_l}\ln\frac{\lambda}{\delta\nu_m}}{\ln\frac{\lambda}{a}\ln^2\frac{\lambda}{\delta\nu_l}};$$

$$\frac{a^2}{\delta^2} \ll \nu^2_{l,m} \ll \frac{\lambda^2}{\delta^2}. \qquad (19)$$

It can be seen from these formulas that the coefficients of the inter-transformation of the magnetohydrodynamic waves R_{l_m} decrease with an increase in the wave number and a decrease in the magnetic field strength. If the plasma wave guide is surrounded by a conducting casing $r = b$, $-\infty < z < +\infty$, it is impossible to study magnetohydrodynamic waves in the case of $\lambda \gg b$, and only the transformation of one oscillation into another occurs at the end of the wave guide. The transformation coefficients for this case may determined by substituting $X^+(t)$ from the solution of the corresponding problem $X + (t) = \left[va\ln\frac{a}{b}\right] X - (t)$ and the wave numbers $h_{\ell,m}$ -- from the solution of the dispersion equation, in expression(15).

$$D_1(h_l) = Z_l^1 - \frac{i\left(h_l^2 - k^2\right)a}{k}\ln\frac{a}{b} = 0.$$

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DETERMINATION OF PLASMA TEMPERATURE AND DENSITY DISTRIBUTION BY REFRACTION AND DAMPING OF A BEAM

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Let us investigate the passage of a beam through a nonuniform plasma. We shall show that, by changing the angle of incidence or the frequency of the microwave signal, we can determine the plasma density distribution by the beam refraction, and can determine the plasma electron temperature distribution by the damping of the wave energy along the beam.

In order to determine the beam trajectory in a plasma, let us employ the Fermi principle

$$\delta \int_a^b k\,ds = 0, \qquad (1)$$

where $\vec{k}$ is the wave vector; ds -- an element of length along the beam trajectory. In an isotropic plasma, we have

$$k = \frac{1}{c}\sqrt{\omega^2 - \omega_0^2}, \qquad (2)$$

where ω is the wave frequency; $\omega_0(\vec{r}) = \sqrt{\frac{4\pi^2 n(\vec{r})}{m}}$ -- plasma Langmuir frequency; $n(\vec{r})$ -- plasma electron density. Modulation is performed in formula (1) for fixed ends ($\delta x_i|_a = \delta x_i|_b = 0$. Taking the fact into account that $ds = \sqrt{g_{ik}dx_i dx_k}$, from expression (1), with allowance for (2), we may obtain the equation which determines the beam trajectory for given $\omega_0(\vec{r})$ /110

$$\frac{d}{ds}\left(g_{ik}\sqrt{\omega^2-\omega_0^2}\,\frac{dx_k}{ds}\right)-\frac{1}{2}\sqrt{\omega^2-\omega_0^2}\,\frac{\partial g_{kl}}{\partial x_i}\cdot\frac{dx_k}{ds}\cdot\frac{dx_l}{ds}-\frac{\partial}{\partial x_i}\sqrt{\omega^2-\omega_0^2}=0. \qquad (3)$$

By employing equation (3), let us investigate the inverse problem: Let us find $\omega_0(\vec{r})$, i.e., the plasma density distribution based on the beam refraction, as a function of the angle of incidence for a fixed frequency, or as a function of the frequency for a fixed angle of incidence.

Planar Problem

If the density of a plasma filling the halfspace $x > 0$ depends only on one coordinate x, and it increases monotonically with an increase in x, then the beam trajectory in the case under consideration is flat (Figure 1). Equation (3) may be written as follows

$$\frac{d}{ds}\left(\sqrt{\omega^2-\omega_0^2}\,\frac{dz}{ds}\right)=0;\ ds=\sqrt{dx^2+dz^2}. \qquad (4)$$

Integrating equation (4), we obtain

$$\sqrt{\omega^2-\omega_0^2}\,\frac{dz}{ds}=\alpha=\text{const}, \qquad (5)$$

where $\alpha = \omega \sin \psi$, ψ is the angle of incidence.

The position of the point at which the beam leaves the plasma $z = \ell$ can be determined from the plasma density distribution; it depends on the angle of incidence and frequency. If the dependence $\ell(\psi)$ is known from experiments, we may obtain the density distribution n(x) in the case of $x < x^*$, where $\omega_0(x^*) = \omega$ (this problem is similar to the problem of determining the potential energy by the specific dependence of the oscillation period on energy [Ref. 1]). In actuality, it follows from equation (5) that

$$\frac{dz}{dx}=\frac{\sin\psi}{\sqrt{\cos^2\psi-\frac{\omega_0^2}{\omega^2}}}. \qquad (6)$$

We thus have

$$l(\psi)=2\int_0^{x_0}\frac{\sin\psi\,dx}{\sqrt{\cos^2\psi-\frac{\omega_0^2}{\omega^2}}}, \qquad (7)$$

/111

where x = x_0 is the rotation point of the beam, determined from the following condition

$$\cos^2\psi = \frac{\omega_0^2(x_0)}{\omega^2}.$$

Introducing a new variable $u = \frac{\omega_0^2}{\omega^2}$ instead of x, from formula (7) we obtain

$$l(\psi) = 2\int_0^{\cos^2\psi} \frac{\sin\psi \frac{\partial x}{\partial u}\,du}{\sqrt{\cos^2\psi - u}}. \tag{8}$$

It thus follows that

$$\int_0^{\gamma} \frac{l(\psi)\,d\cos^2\psi}{\sin\psi\sqrt{\gamma-\cos^2\psi}} = +2\int_0^{\gamma} d\cos^2\psi \int_0^{\cos^2\psi} \frac{\frac{\partial x}{\partial u}\,du}{\sqrt{(\cos^2\psi - u)(\gamma-\cos^2\psi)}} =$$

$$= +2\pi\int_0^{\gamma}\frac{\partial x}{\partial u}du = +2\pi x(\gamma).$$

Assuming that $\gamma = \frac{\omega_0^2}{\omega^2}$, we finally find that

$$x = \frac{1}{2\pi}\int_0^{\frac{\omega_0^2}{\omega^2}} \frac{l(\psi)\,d\cos^2\psi}{\sin\psi\sqrt{\frac{\omega_0^2}{\omega^2} - \cos^2\psi}}. \tag{9}$$

Equation (9) determines the function $x(\omega_0)$, i.e., the dependence of density on the coordinate in implicit form, according to the specific dependence $\ell(\psi)$. We may employ this equation in the case of a cylindrical plasma, when the plasma is probed by a beam in the plane ϕ = const passing through the cylinder axis. Thus, $\omega_0 = \omega_0(r)$ and x = r = R, where R is the plasma radius ($\omega_0(R) = 0$). By performing similar measurements for different values of the aximuthal angle ϕ, we may obtain the density distribution in the case when density depends on the angle ϕ (but does not depend on z). /112

The plasma density distribution may also be determined by probing the plasma with microwave signals having a different frequency. For a given angle of incidence, the position of the exit point of the beam $z = \ell$ depends on the frequency ω. Knowing the function $\ell(\omega)$ from experiment, we

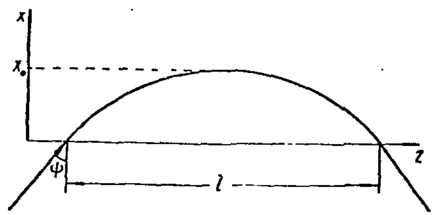

Figure 1

may readily determine the function ω_0 (x) according to equation (7). It follows from equation (7) that

$$\int_0^{\gamma} \frac{l(\omega)\, d\omega}{\operatorname{tg}\psi \sqrt{\gamma^2 - \omega^2}} = \int_0^{\gamma} d\omega \int_0^{x_0} \frac{2\omega dx}{\sqrt{(\gamma^2 - \omega^2)\left(\omega^2 - \frac{\omega_0^2}{\cos^2\psi}\right)}} = \pi x(\gamma). \tag{10}$$

Assuming that $\gamma = \frac{\omega_0}{\cos\psi}$, we obtain the expression determining the dependence $\omega_0(x)$ in implicit form

$$x = \frac{1}{\pi \operatorname{tg}\psi} \int_0^{\frac{\omega_0}{\cos\psi}} \frac{l(\omega)\, d\omega}{\sqrt{\omega_0^2/\cos^2\psi - \omega^2}}. \tag{11}$$

We may also obtain formula (11) from the expression given in (Ref. 3) [see also (Ref. 4)] for the real layer height $x(\omega_0)$ according to the specific effective height $x_g(\omega)$ for the case of normal wave incidence on the layer, with allowance for the following relationship (Ref. 4)

$$l(\psi) = 2\operatorname{tg}\psi x_g(\omega\cos\psi).$$

Cylindrical Problem

If the beam passes through a plasma cylinder in a plane which is perpendicular to the cylinder axis (Figure 2), equation (3) may be represented in the following form

$$\frac{d}{ds}\left(r^2 \sqrt{\omega^2 - \omega_0^2}\, \frac{d\varphi}{ds}\right) = 0;\ ds = \sqrt{dr^2 + r^2 d\varphi^2}. \tag{12}$$

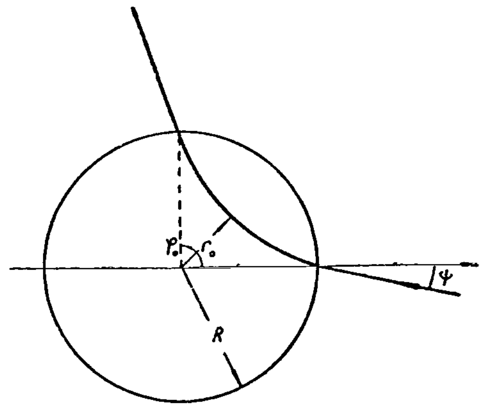

Figure 2

The plasma density distribution n(r) may be found from the specific experimental dependence of the angle at which the beam leaves the plasma ϕ_0 on the angle of incidence ψ * (this problem is similar to the problem of determining the potential energy from the specific dependence /113 of the scattering cross section on the scattering angle (Ref. 1, 2). Integrating equation (12), we obtain

$$\frac{dr}{d\varphi}=\frac{r^2}{R\sin\psi}\sqrt{1-\frac{\omega_0^2}{\omega^2}-\frac{R^2}{r^2}\sin^2\psi}. \tag{13}$$

It thus follows that

$$\varphi_0(\psi)=2R\sin\psi\int\limits_{r_0}^{R}\frac{dr}{r^2\sqrt{1-\frac{\omega_0^2}{\omega^2}-\frac{R^2}{r^2}\sin^2\psi}}, \tag{14}$$

where $r = r_0$ is the rotation point of a beam which is determined from the following condition

$$\frac{\omega_0^2(r_0)}{\omega^2}+\frac{R^2}{r_0^2}\sin^2\psi=1.$$

Assuming that

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* This problem was solved in (Ref. 5).

$$\tau = \ln\frac{R}{r}; \; u = \left(1 - \frac{\omega_0^2}{\omega^2}\right)\frac{r^2}{R^2},$$

we can write equation (14) in the following form

$$\varphi_0(\psi) = 2\sin\psi \int_1^{\sin^2\psi} \frac{\frac{\partial\tau}{\partial u}\,du}{\sqrt{u - \sin^2\psi}}, \tag{15}$$

from which we find that

$$\int_\gamma^1 \frac{\varphi_0(\psi)\,d\sin\psi}{\sqrt{\sin^2\psi - \gamma^2}} = 2\int_\gamma^1 \sin\psi d\sin\psi \int_1^{\sin^2\psi} \frac{\frac{\partial\tau}{\partial u}\,du}{\sqrt{(\sin^2\psi - \gamma^2)(u - \sin^2\psi)}} = \pi\tau(\gamma).$$

Assuming that $u = \gamma^2 = \left(1 - \frac{\omega_0^2}{\omega^2}\right)\frac{r^2}{R^2}$, we may find the dependence $r = r(\omega_0)$:

$$r = R\exp\left\{-\frac{1}{\pi}\int_{\sqrt{u}}^1 \frac{\varphi_0(\psi)d\sin\psi}{\sqrt{\sin^2\psi - u}}\right\}. \tag{16}$$

Formula (16) determines the function $r = r(\omega_0)$, i.e., the distribution of density over the radius in implicit form, according to the specific dependence of the beam exit angle on the angle of incidence. /114

Performing measurements in different planes z = const, we may thus obtain the density distribution along the cylinder axis in the case of axial symmetry. For a fixed angle of incidence ψ, equation (15) determines the beam exit angle ϕ_0 as a function of frequency. A knowledge of the function $\phi_0(\omega)$ enables us to determine the density distribution. We thus assume

$$\Omega(r) = \frac{\omega_0(r)}{\sqrt{1 - \frac{R^2}{r^2}\sin^2\psi}},$$
$$\xi = \arcsin\left(\frac{R}{r}\sin\psi\right). \tag{17}$$

Equation (14) may then be written as

$$\frac{\varphi_0(\omega)}{2\omega} = \int_0^{\omega} \frac{\frac{\partial \xi}{\partial \Omega} d\Omega}{\sqrt{\omega^2 - \Omega^2}}.$$

We thus find

$$\int_0^{\gamma} \frac{\varphi_0(\omega)\, d\omega}{\sqrt{\gamma^2 - \omega^2}} = \int_0^{\gamma} d\omega \int_0^{\omega} \frac{2\omega \frac{\partial \xi}{\partial \Omega} d\Omega}{\sqrt{(\omega^2 - \Omega^2)(\gamma^2 - \omega^2)}} = \pi\, [\xi(\gamma) - \xi(0)].$$

Since $\xi(0) \equiv \xi\big|_{\Omega=0} = \psi$, assuming that $\gamma = \Omega$, we can write

$$\xi(\Omega) = \psi + \frac{1}{\pi} \int_0^{\Omega} \frac{\varphi_0(\omega)\, d\omega}{\sqrt{\Omega^2 - \omega^2}}.$$

We thus obtain the expression determining the dependence $\Omega(r)$ in implicit form

$$r = \frac{R \sin \psi}{\sin \left\{ \psi + \frac{1}{\pi} \int_0^{\Omega} \frac{\varphi_0(\omega)\, d\omega}{\sqrt{\Omega^2 - \omega^2}} \right\}}. \tag{18}$$

Temperature Determination from Beam Damping

Due to collisions, the energy of a beam leaving the plasma is e^{τ} times less than the incident energy, where $t = \int \varkappa\, ds$ is the optical plasma thickness; $\varkappa$ -- damping coefficient. If the wave frequency is considerably greater than the frequency of collisions of electrons with ions and neutral particles $\nu(r)$, we then have /115

$$\varkappa(r) = \frac{\nu(r)\, \omega_0^2(r)}{c\omega^2 \sqrt{1 - \frac{\omega_0^2(r)}{\omega^2}}}.$$

Knowing $\varkappa(r)$, we may find the plasma temperature distribution from the specific dependence of ν on temperature.

The damping τ depends on the angle of incidence ψ. We may find the dependence of the damping coefficient on the radius from the quantity $\tau(\psi)$, which is measured experimentally, and from the specific density distribution.

Let us first investigate the planar problem (in the case of a cylindrical plasma, the beam trajectories lie in the plane ϕ = const). For

$\tau(\psi)$, we may write the following expression

$$\tau(\psi)=2\int_0^{x_0} \varkappa(x) \sqrt{1+\left(\frac{dz}{dx}\right)^2}\, dx.$$

Substituting $\frac{dz}{dx}$ from formula (5) in this, we obtain

$$\tau(\psi)=2\int_0^{x_0} \varkappa(x) \sqrt{\frac{\omega^2-\omega_0^2}{\omega^2\cos^2\psi-\omega_0^2}}\, dx = 2\int_0^{\cos^2\psi} \varkappa \sqrt{\frac{1-u}{\cos^2\psi-u}}\, du. \tag{19}$$

Let us introduce the function f(u) according to the equation

$$\frac{df}{du}=\varkappa(u)\sqrt{1-u}\,\frac{dx}{du}. \tag{20}$$

We then have

$$\tau(\psi)=2\int_0^{\cos^2\psi} \frac{\frac{\partial f}{\partial u}\, du}{\sqrt{\cos^2\psi-u}}. \tag{21}$$

Solving this equation and equation (8), we find

$$\frac{df(\lambda)}{d\lambda}=\frac{1}{2\pi}\int_0^{\lambda} \frac{\frac{\partial\tau}{\partial\cos^2\psi}\, d\cos^2\psi}{\sqrt{\lambda-\cos^2\psi}}.$$

Assuming that $\lambda = \frac{\omega_0^2}{\omega^2}$, we obtain /116

$$\varkappa(x)=\frac{1}{\pi}\left(\frac{d}{d\tau}\sqrt{1-\frac{\omega_0^2}{\omega^2}}\right)\int_0^{\frac{\omega_0}{\omega}} \frac{\frac{\partial\tau}{\partial\cos\psi} d\cos\psi}{\sqrt{\frac{\omega_0^2}{\omega^2}-\cos^2\psi}}. \tag{22}$$

In explicit form, this formula determines the dependence of the damping coefficient on coordinate x.

If the beam trajectories lie in the z = const plane, we then have

$$\tau(\psi)=2\int_{r_0}^{R} \varkappa(r)\sqrt{1+r^2\left(\frac{d\varphi}{dr}\right)^2}\, dr.$$

Substituting the value of $\frac{d\phi}{dr}$ from formula (13), we obtain

$$\tau(\psi)=2\int_{r_0}^{R}\varkappa(r)\sqrt{\frac{\omega^2-\omega_0^2}{\omega^2-\omega_0^2-\omega^2\frac{R^2}{r^2}\sin^2\psi}}\,dr. \tag{23}$$

Assuming that

$$u=\frac{r^2}{R^2}\left(1-\frac{\omega_0^2}{\omega^2}\right);\quad \frac{df(u)}{du}=\varkappa(r)\frac{r}{R}\sqrt{1-\frac{\omega_0^2}{\omega^2}}\cdot\frac{dr}{du},$$

we may write equation (23) in the following form

$$\tau(\psi)=2\int_{\sin^2\psi}^{1}\frac{\frac{df}{du}\,du}{\sqrt{u-\sin^2\psi}}. \tag{24}$$

Solving it in the same way as equation (19), we find

$$\frac{df(\gamma)}{d\gamma}=\frac{1}{2\pi}\int_{\gamma}^{1}\frac{\frac{\partial\tau}{\partial\sin^2\psi}\,d\sin^2\psi}{\sqrt{\sin^2\psi-\gamma}}.$$

Assuming that $\gamma = u(r)$, we obtain the following expression /117

$$\varkappa(r)=\frac{1}{\pi}\left|\frac{d\sqrt{u}}{dr}\int_{\sqrt{u}}^{1}\frac{\frac{\partial\tau}{\partial\sin\psi}\,d\sin\psi}{\sqrt{\sin^2\psi-u}}\right|_{u=\frac{r^2}{R^2}\left(1-\frac{\omega_0^2}{\omega^2}\right)}. \tag{25}$$

Expression (25) determines the dependence of the damping coefficient coefficient on the radius according to the specific dependence of τ on ψ.

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SECTION III

PLASMA NONLINEAR OSCILLATIONS AND WAVE INTERACTION /118

EXCHANGE OF ENERGY BETWEEN HIGH FREQUENCY AND LOW FREQUENCY OSCILLATIONS IN A PLASMA

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Oscillations with frequencies ν on the order of 100 kc (Ref. 1), as well as oscillations at the electron plasma frequency ($\omega_{0e} \approx 50$ Mc), occur in a plasma produced by an electron bundle in a longitudinal magnetic field at a pressure of 10^{-6} — 10^{-5} mm Hg. It has been found that there is a relationship between the low frequency and high frequency oscillations (Ref. 1 - 3). High frequency oscillations cause an increase in the low frequency oscillation amplitude. On the other hand, they undergo amplitude modulation at a low frequency, which leads to the occurrence of combined frequencies $\omega \pm \nu$. The amplitudes of oscillations having the frequencies $\omega + \nu$ and $\omega - \nu$ are not the same, which -- just as with the Landsberg-Mandel'shten-Raman effect -- can indicate the direction and effectiveness of energy transfer from oscillations of one frequency to oscillations of another frequency.

As has been shown previously (Ref. 1), low frequency oscillations /119 represent ion motion (which is transverse with respect to the bundle) in the field of the space charge of electrons contained by a magnetic field. The frequency of these oscillations may be readily computed from the following simple model. Let us assume that electrons and ions fill one and the same cylindrical region in the equilibrium state. The simultaneous shift of all ions by the quantity $\vec{r}_i$, and of electrons by the quantity $\vec{r}_e$, leads to polarization, which can be determined as follows in the case of a small shift

$$\vec{E} = -\frac{ne}{\varepsilon_0}(\vec{r}_i - \vec{r}_e), \tag{1}$$

where n is the plasma density.

We can write the following expression for electrons and ions

$$\ddot{\vec{r}}_i = \frac{e}{m_i}\vec{E} + \frac{e}{m_i}[\dot{\vec{r}}_i\vec{B}]; \tag{2}$$

$$\ddot{\vec{r}}_e = -\frac{e}{m_e}\vec{E} - \frac{e}{m_e}[\dot{\vec{r}}_e\vec{B}] \tag{3}$$

or, with allowance for (1),

$$\ddot{\vec{r}}_i = -\omega_{0i}^2(\vec{r}_i - \vec{r}_e) + \frac{e}{m_i}[\dot{\vec{r}}_i\vec{B}]; \tag{4}$$

$$\ddot{\vec{r}}_e = \omega_{0e}^2(\vec{r}_i - \vec{r}_e) - \frac{e}{m_e}[\dot{\vec{r}}_e\vec{B}], \tag{5}$$

where

$$\omega_{0e}^2 = \frac{ne^2}{\varepsilon_0 m_e}; \tag{6}$$

$$\omega_{0e}^2 = \frac{ne^2}{\varepsilon_0 m_i}. \tag{7}$$

Disregarding the electron shift during orbital motion, we shall only allow for their drift in crossed electric and magnetic fields, and we shall designate the position of the electron guiding center by the vector $\vec{r}_e$. According to expression (5), we then have

$$\vec{r}_i - \vec{r}_e = \frac{e}{m_e\omega_{0e}^2}[\dot{\vec{r}}_e\vec{B}] = \frac{e}{m_i\omega_{0i}^2}[\dot{\vec{r}}_e\vec{B}]. \tag{8}$$

It follows from formulas (4) and (8) /120

$$\ddot{\vec{r}}_i = -\frac{e}{m_i}[\dot{\vec{r}}_e\vec{B}] + \frac{e}{m_i}[\dot{\vec{r}}_i\vec{B}],$$

and thus

$$\dot{\vec{r}}_i = -\frac{e}{m_i}[\vec{r}_e\vec{B}] + \frac{e}{m_i}[\vec{r}_i\vec{B}] = \frac{e^2}{m_i\omega_{0i}^2}[[\dot{\vec{r}}_e\vec{B}]\vec{B}] = -\frac{e^2B^2}{m_i\omega_{0i}^2}\dot{\vec{r}}_e \tag{9}$$

or

$$\dot{\vec{r}}_e = \frac{\omega_{0i}^2}{\omega_{Hi}^2}\vec{r}_i = -\Omega_H^{-2}\vec{r}_i, \tag{10}$$

where

$$\Omega_H = \frac{\omega_{Hi}}{\omega_{0i}} \tag{11}$$

-- the ratio between the ion cyclotron frequency and the plasma frequency.

Substituting the expression obtained for $\vec{r}_e$ in formula (4), we obtain

$$\ddot{\vec{r}}_i - \frac{e}{m_i}[\dot{\vec{r}}_i\vec{B}] + \omega_{0i}^2(1 + \Omega_H^{-2})\vec{r}_i = 0. \tag{12}$$

Equation (12) has a $\vec{r}_i \sim e^{i\nu t}$ type of solution, and the frequency must satisfy the following equation

$$\Omega^2 + \Omega_H\Omega - 1 - \Omega_H^{-2} = 0, \tag{13}$$

i.e.,

$$\Omega_{12} = -\frac{\Omega_H}{2} \pm \sqrt{\frac{\Omega_H^2}{4} + 1 + \Omega_H^{-2}}. \tag{14}$$

In the case of $\Omega_H \approx 1$, $\nu \approx \omega_{0i} \approx \sqrt{\frac{n}{m_i}}$.

If $\Omega_H >> 1$, equation (13) has two roots:

$$\Omega_1 = -\Omega_H \tag{15}$$

$$\Omega_2 = \frac{1}{\Omega_H}. \tag{16}$$

If $\Omega_H << 1$

$$\Omega = \pm\frac{1}{\Omega_H}. \tag{17}$$

The oscillation frequency ν decreases with an increase in the magnetic field strength, which coincides with the solutions of (16) and (17), if it is assumed that the frequency ω_{0i} depends slightly on the magnetic field strength. If ω_{0i} does not depend on B -- i.e., ω_{0i} is constant -- it may /121 be readily determined according to any pair of measured ν and B, which would then enable us to compile a graph showing the dependence of ν on B according to formula (14). The dependence of ion oscillation frequency on the magnetic field strength is shown in Figure 1. The curves were recorded for different current values in an electron bundle (1 - I = 10 ma, 2 - I = 30 ma). The dashed curves are drawn through the points computed according to formula (14), under the assumption that the plasma density does not change when there is a change in the magnetic field strength. The computed points do not lie on the experimental curves (particularly in the region of small fields), which is no doubt related to the variability of ω_{0i}. Nevertheless, it may be stated that the nature of the dependence is correctly imparted by our simple model.

Modulation of high frequency oscillations may be due to low frequency oscillations of the plasma density. Let us assume, for example, that the plasma is located in an external field $\vec{E}e^{i\omega t}$, which is transverse with respect to the bundle, and the frequency ω is so great that only allowance for the electron component shift has any meaning. The external field $\vec{E}e^{i\omega t}$ gives rise to a shift and the occurrence of a polarized field

$\sim \frac{ne}{\varepsilon_0}\vec{r}_\sim$. The equation for electron motion in these fields may be written in the following form

$$\ddot{\vec{r}}_\sim = -\frac{e}{m_e}\left(\vec{E}e^{i\omega t} + \frac{ne}{\varepsilon_0}\vec{r}_\sim\right) - \frac{e}{m_e}[\dot{\vec{r}}_\sim \vec{B}]. \tag{18}$$

We obtain the following expression for forced electron oscillations

$$\vec{r}_\sim = \frac{-\frac{e}{m_e}\vec{E}e^{i\omega t}}{\omega_{0e}^2 + \omega(\omega_{He} - \omega)}. \tag{19}$$

If it is assumed that the plasma density undergoes oscillations at the frequency $\nu \ll \omega$ and the quantity ω_{0e}^2 has the form $\omega_{0e}^2(1 + \alpha \cos \nu t)$, while $\alpha \ll 1$, then expression (19) changes into the sum of the oscillations with the principal frequencies (ω) and the combined frequencies ($\omega \pm \nu$): /122

$$\vec{r}_\sim = \frac{e\vec{E}e^{i\omega t}}{m_e\omega_{0e}^2\left[1 + \frac{\omega}{\omega_{0e}^2}(\omega_{He} - \omega)\right]} + \frac{\alpha}{2}\cdot\frac{e}{m_e}\cdot\frac{\vec{E}}{\omega_{0e}^2}\cdot\frac{e^{i(\omega+\nu)t}}{\left[1 + \frac{\omega}{\omega_{0e}^2}(\omega_{He} - \omega)\right]^2} + \frac{\alpha}{2}\cdot\frac{e}{m_e}\cdot\frac{\vec{E}}{\omega_{0e}^2}\cdot\frac{e^{i(\omega-\nu)t}}{\left[1 + \frac{\omega}{\omega_{0e}^2}(\omega_{He} - \omega)\right]^2}. \tag{20}$$

The oscillations have resonance close to the electron cyclotron frequency.

This model is inadequate for determining oscillation intensity at the combined frequencies.

The experiments were performed on a hollow electron bundle in a longitudinal magnetic field with a strength ranging between 200 - 2000 oersted. The bundle length was 50cm; diameter -- 2 cm; energy -- 250 v; and the current -- 20 - 40 ma. The bundle was located in a metallic tube having a diameter of 9 cm. Pressure in the chamber was several units of 10^{-6} mm Hg. The interaction between the outer, high frequency field and the low frequency ion oscillations was studied. The interaction was observed in three cases: When the frequency of the outer signal (1) did not coincide with any of the plasma eigen frequencies, (2) coincided with the electron cyclotron frequency, and (3) coincided with the plasma electron frequency.

In the nonresonance case, the outer field was transverse with respect to the bundle. It was produced between two conductors having a diameter of 0.2 cm and a length which was close to the bundle length. The conductors were parallel to the bundle axis at a distance of 6.5 cm from each other. Figure 2 shows the diagram of the experimental apparatus employed to /123

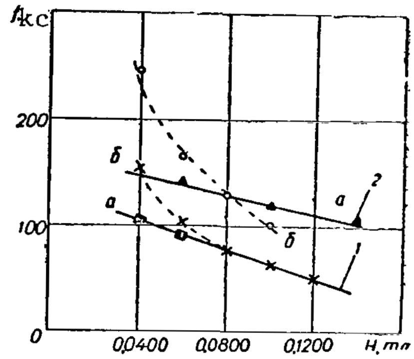

Figure 1

study the interaction between ion oscillations and the outer electric field, whose frequency does not coincide with any of the plasma eigen frequencies (1 -- solenoid; 2 -- electron gun; 3 -- probe; 4 -- hollow electron bundle; 5 -- collector; 6 -- conductors between which a high frequency field is produced; 7 -- coils connecting the generator and the measurement circuit. The resonance curve for the circuit producing the high frequency field is shown in the upper right). The capacitor is a section of the resonance circuit weakly connected to the GS-23 generator and the C4-8 spectrum analyzer. The resonance frequency of the circuit was 13.88 Mc. The resonance barely shifted when the bundle was switched on. The resonance width was quite large (Q = 50), so that the combined frequencies did not go beyond it. The connection between the circuit, the generator, and the measurement circuit was selected so that the resonance curve was symmetrical. This was important for comparing the intensities of combined oscillations with the frequencies $\omega + \nu$ and $\omega - \nu$.

Figure 3 presents typical spectra of oscillations produced when the resonance frequency was used (the spectra were obtained under the following conditions: Pressure $p = 3.4 \cdot 10^{-6}$ mm Hg, electron bundle current I = 30 ma, anode voltage U = 250 v, effective variable outer field strength $v_{\sim} = 64$ v, magnetic field strength H: a -- 600 oersted; b -- 780 oersted; c -- 920 oersted). Lateral frequencies spaced at the frequency of ion oscillations may be seen, in addition to the frequency employed. Employing the terminology used in the theory of combined scattering, we shall call the lateral lines red ($\omega - \nu$) and violet ($\omega + \nu$) "companions". The relative height of the companions (with respect to the carrier height) was 1 - 2%, and it increased with an increase in the amplitude of ion oscillations and the amplitude of the outer signal. /124

The heights of the red and violet companions, generally speaking, were different, and this difference depended on the magnetic field

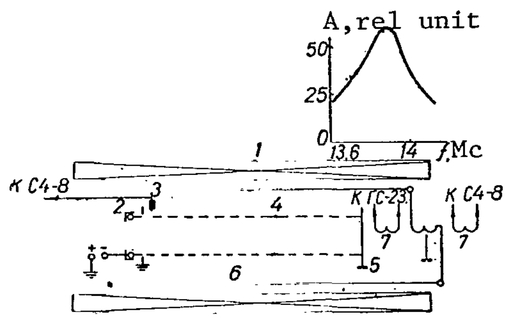

Figure 2

strength. As may be seen from Figure 3, when the magnetic field strength was 600 oersteds, the violet companion was higher, and for 920 oersteds the red companion was higher. At a field strength of 780 oersteds, the companions were the same. In order to explain the difference in the heights, we should point out that the increase in the ion oscillation amplitude when a high frequency outer signal was employed was always greater for large magnetic fields. For small magnetic fields and the same intensity of the outer signal, the amplitude increase in ion oscillations disappeared. Thus, an increase in the magnetic field strength can improve the conditions for transferring energy from high frequency oscillations to ion oscillations.

Comparing this with the data given in Figure 3, we may arrive at the conclusion that there is a relationship between the companion heights and the direction of energy transfer. The predominance of the red companion corresponds to the transfer of energy from high frequency oscillations to low frequency oscillations; the predominance of the violet companion corresponds to the energy transfer in the opposite direction. The direction of energy transfer no doubt depends on the relationship between the amplitudes of the ion oscillations and the outer signal. In actuality, an increase in the magnetic field strength decreases the ion oscillation amplitude, which leads to a more effective energy transfer from the high frequency to the low frequency for a given amplitude of the outer signal.

The same result may be achieved in another way: by changing the outer signal amplitude for a constant magnetic field. With an increase in the outer signal, the red companion becomes higher as compared with the violet. /125

Let us turn to an experiment in which the outer field frequency coincides with the electron cyclotron frequency. Figure 4 shows the

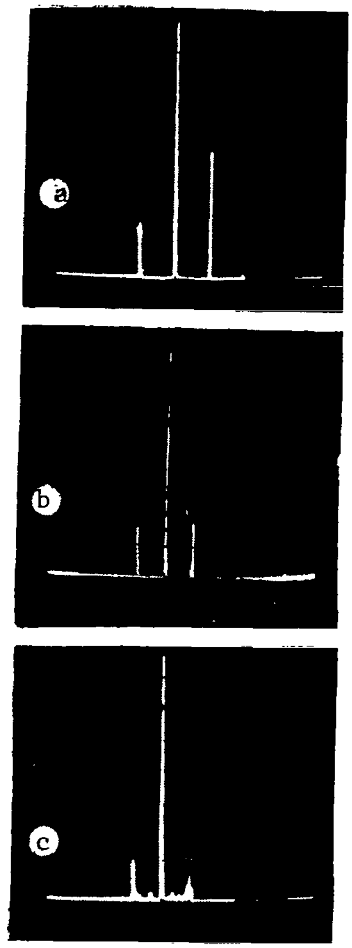

Figure 3

diagram of the experimental apparatus for studying the interaction between ion oscillations and the outer field at the electron cyclotron frequency (1 -- solenoid; 2 -- electron gun; 3 -- probe; 4 -- hollow electron bundle; 5 -- collector; 6 -- volumetric resonator; 7 -- connection with the oscillation source and measurement circuit). The outer field was produced in the volumetric resonator which encompassed almost all of the bundle. The mode H_{11} was excited in the resonator at a frequency of 2265 Mc. When the bundle was switched on, the oscillation level in the resonator sharply decreased, when the magnetic field strength reached a value corresponding to electron cyclotron resonance (Figure 5). The width of the resonance absorption curve was primarily determined by the

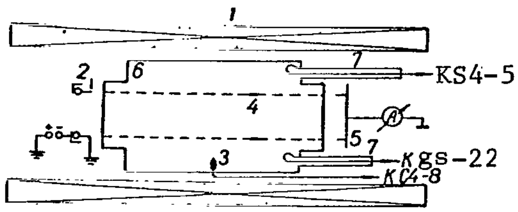

Figure 4

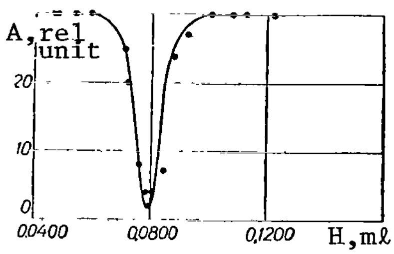

Figure 5

nonuniformity of the magnetic field over the length of the system. Companions appeared close to resonance which coincided with formula (20). The violet companion was higher than the red companion, which was apparently related to the small amplitude of the signal supplied.

Let us examine the case when the outer field frequency coincides with the electron plasma frequency. The outer signal is supplied to a grid located at the bundle origin. Measurements are performed by a probe. Figure 6 shows the spectra for several frequencies close to the electron plasma frequency ($p = 4 \cdot 10^{-6}$ mm Hg, I = 40 ma, U = 250 v, H = 1000 oersted, voltage on the grid $u_c = 0.1$ v. Oscillation frequencies of voltage on the grid: a -- 39 Mc; b -- 40 Mc; c -- 41 Mc; d -- 42 Mc; e -- 43 Mc). It can be seen that the interaction is resonant in nature. The heights of the companions are large (the total altitude of the carrier is shown in the photographs), which points to the effectiveness of the interaction between ion oscillations and the outer signal at the electron plasma frequency. /126

The amplitude of the signal supplied is 0.1 v *, which explains the

* The effective voltages are always employed.

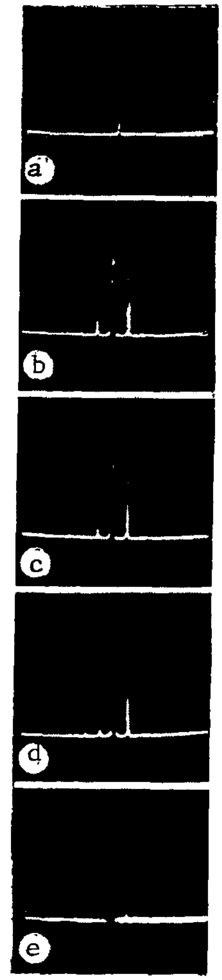

Figure 6

predominance of the violet companion. When the amplitude of the outer high frequency field increases (for a magnetic field strength of 1000 oersted), the red companion begins to predominate. A similar relationship between the companion heights and the amplitude of the outer field is observed for H = 7000 oersted. With an increase in the red companion, the altitude of the principal line (carrier) decreases. If H = 700 oersted, a decrease in even the principal line may be observed when the outer signal is intensified (from $u_c = 0.5$ v to $u_c = 0.95$ v). For a comparatively small magnetic field (H = 400 oersted) and a significant outer signal amplitude, there is very strong interaction which is accompanied by the appearance of many combined frequencies.

These data point to the effective transfer of energy from electron plasma oscillations to ion oscillations. This phenomenon may probably be employed to increase the energy of the plasma ion component.

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DISSIPATION OF PLASMA OSCILLATIONS EXCITED IN A CURRENT-CARRYING PLASMA

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Several experimental and theoretical investigations have studied the development of bunched instabilities in a current-carrying plasma for large electric field strengths (Ref. 1 - 5). It has been shown that, as only the mean energy of the ordered electron drift is larger than their thermal energy, intense longitudinal electrostatic oscillations develop /127 in a plasma. Their energy reaches the initial energy level of electron drift usually after several tens of plasma oscillation periods.

Computations of the multi-flux motion of electrons in a current-carrying plasma, which were performed by O. Buneman (Ref. 6), J. Dawson (Ref. 7), and Ya. B. Faynberg (Ref. 8), have shown that very intense "thermalization" of the plasma oscillation energy occurs, if this energy is considerably greater than the electron thermal energy. Thermalization occurs due to nonlinear phenomena leading to the transformation of longitudinal oscillations into transverse oscillations and to their rapid phase mixing.

This process takes place until the energy of the ordered oscillations equals the electron thermal energy. It may be assumed that ion heating will occur due to "collective" friction of electrons on ions in the case of bunched instabilities.

Thus, an investigation of bunched instabilities in a high-current gas discharge opens up new possibilities for effective plasma heating.

The studies (Ref. 9, 10) performed detailed investigations of the excitation conditions of bunched instabilities in a current-carrying plasma, as well as the plasma characteristics in the presence of these instabilities. The occurrence of an anomalously high discharge resistance and intense microwave plasma radiation was discovered.

This article investigates heating and containment of a plasma in a strong magnetic field, under conditions when bunched instabilities excited by "escaping" electrons develop in the plasma. The experiments were performed on an apparatus representing a rectilinear tube made of alundum with a diameter of 10 cm and a length of 25 cm, which was usually filled with hydrogen at a pressure of $5 \cdot 10^{-3}$ - 10^{-4} mm Hg. Aluminum electrodes were placed at the two ends of the tube; a battery of capacitors having an over-all capacitance of 15 microfarads was discharged between the electrodes. The battery was charged to a voltage of 30-40 kv. The discharge current through the gas amounted to 100 ka with a period of 9 microseconds. In order to eliminate hydromagnetic phenomena, the discharge was performed in a strong longitudinal magnetic field (on the order of 1.2 tl), at which the Shafranov condition of stability would be fulfilled (Ref. 13). In order that the plasma did not touch the walls, a diaphragm with an opening which was 80 mm in diameter was placed between the electrodes.

During the first half-period in the discharge, a highly ionized plasma filament, which was separated from the wall and which had a diameter of 80 mm, was produced; no macroscale hydromagnetic instabilities were apparent in this plasma filament. The plasma density changed between 10^{14} - 10^{13} cm^{-3}. The construction of the apparatus and the experimental method were described in detail in (Ref. 9). X-ray and microwave radiation from the discharge, the current of "escaping" electrons, the over-all discharge current, and the voltage between the electrodes were studied experimentally. /128

Figure 1 shows the following oscillograms: a -- microwave radiation from the plasma; b -- over-all discharge current; c -- current of "escaping" electrons; d -- voltage between the electrodes of the discharge tube reduced to a single time scale. The oscillograms were recorded at an initial hydrogen pressure in the chamber of $2 \cdot 10^{-2}$ mm Hg, a magnetic field strength of 0.64 tl, and a charge voltage of 34 kv.

Characteristic, inter-correlated oscillations are observed at high

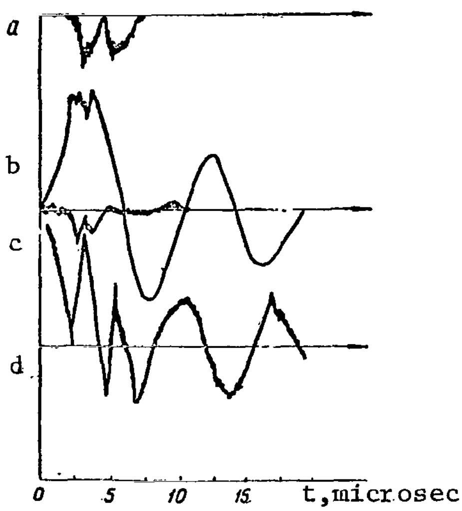

Figure 1

electric field strengths; these oscillations are due to the development of bunched instabilities. During the initial period, the field strength in the discharge center increases as the voltage wave penetrates the plasma. With an increase in the electric field strength in the discharge, accelerating processes begin to develop. A current of "escaping" electrons first appears due to the "tail" of the Maxwell distribution. However, as soon as the electric field strength in the plasma begins to exceed the critical value, all of the electrons acquire a drift velocity which is greater than the thermal velocity, and bunched instabilities develop in the plasma, to which the occurrence of epithermal microwave radiation corresponds. In this case, a large portion of the directional drift energy of the electrons is transmitted to excitation of oscillations, and the current of "escaping" electrons sharply decreases. This leads to an increase /129 in the effective plasma resistance and to a dip on the oscillogram for the over-all discharge current.

The amount of energy contributed by the outer source to the buildup of longitudinal electron oscillations may be computed from the additional current at the moment an instability develops. For the case shown in Figure 1, this energy amounts to 10 kv per particle. The energy of these oscillations considerably exceeds the initial thermal energy equalling 30 electron volts, which leads to effective thermalization of plasma oscillations due to nonlinear phenomena. As a result, the random electron energy will equal, in order of magnitude, the energy of plasma oscillations. Due to thermalization, intense X-ray radiation occurs as a result of energetic electrons falling on the target.

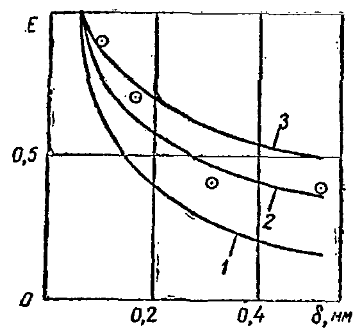

Figure 2

Since the transverse component of electron energy has increased considerably, and the velocity of directed drift has remained the same as previously, the condition for excitation of plasma bunched instabilities has been disturbed and the plasma has returned to the initial unexcited state. Figure 1 clearly illustrates two cycles of such oscillations with a period on the order of 1.5 microseconds. This period is probably determined by the time required for electrons to change into a state of "escape". According to the computations of Dreicer (Ref. 11), this period equals, in order of magnitude, the time between two Coulomb collisions for an electric field strength which is greater than the critical strength. Assuming that 30% of the energy of plasma oscillations is "thermalized" ($T_e = 3 \cdot 10^3$ electronvolts) (Ref. 12), we find that for a plasma density of $7 \cdot 10^{14}$ cm^{-3} the time between two Coulomb collisions is 4.5 microseconds.

The period in which the "heating" cycles are repeated depends on the initial gas pressure in the chamber. It increases considerably with a decrease in the plasma density. The effective electron temperature of the plasma must thus increase, since the total energy transmitted into the buildup of plasma oscillations from the outer source changes very little.

Thus, it would be expected that intense electron heating occurs due to the development of bunched instabilities in the discharge. Direct measurements of the electron temperature are of great interest.

The effective electron temperature was determined by the absorption of electron braking radiation in thin beryllium foils located in front of a scintillation crystal on the wall within the vacuum chamber. The /130 plasma electrons falling on the foil-target are braked in the very thin surface layer. Their energy is transformed into braking X-ray radiation which, after partial absorption in the foil, falls on the crystal causing a flash of light. In order to determine the radiation hardness, without disturbing the vacuum it is possible to place beryllium foils having different thicknesses in front of the crystal. The light from the crystal

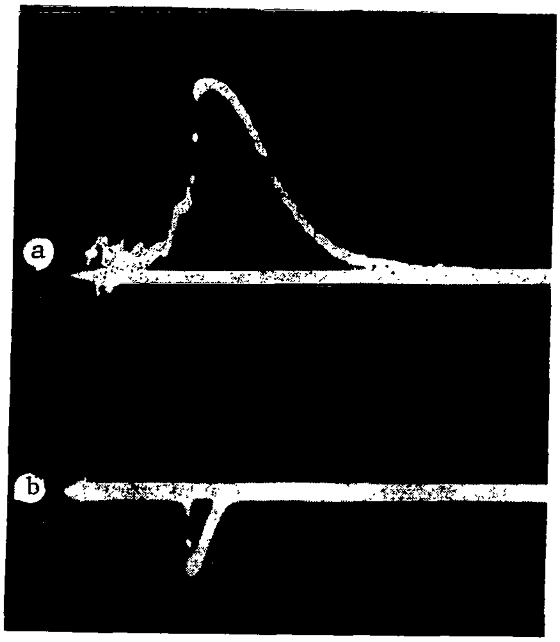

Figure 3

is supplied to the photomultiplier by means of the wave guide. The signal from the photomultiplier through the cathode follower is supplied to the oscillograph amplifier.

Figure 2 shows the dependence of the photomultiplier (PM) signal intensity on the thickness of the absorber foil. This dependence may be employed to determine the electron temperature for a specific form of the electron energy distribution function. Maxwell distribution, rectangular distribution with the width T_e, and Drayvesten distribution lead to similar temperature values. This enables us to employ the curve shown in Figure 2 for a rough estimate of the electron temperature when the electron energy distribution function is not known precisely.

The curves in Figure 2 were compiled under the assumption of Maxwell distribution for three temperatures: 1 -- 1 kev; 2 -- 2 kev; 3 - 3 kev. It can be seen that the experimental points correspond to a plasma electron temperature on the order to 2 kev.

Figure 3 presents the following oscillograms: a -- current of "escaping" electrons; b -- X-ray radiation from the plasma due to braking

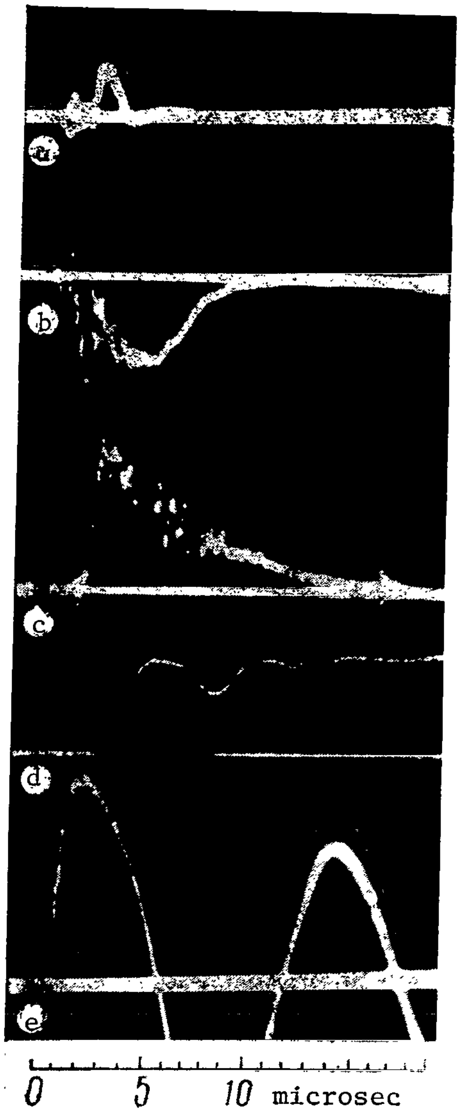

Figure 4

of thermalized electrons in the foil. X-ray radiation arises simultaneously with the current, and continues for a long period of time after the braking of "escaping" electrons due to the energy of transverse motion. The maximum quanta energy of this radiation is about 15 kev. This points to the effective transformation of the energy of electron longitudinal oscillations into the energy of transverse motion. In the absence of bunched instabilities, the electron temperature in the discharge is 30 electronvolts.

Figure 4 presents the following oscillograms: a -- current of "escaping" electrons; b -- X-ray radiation; c -- microwave radiation; d -- light from the discharge center; and e -- over-all discharge current. X-ray radiation arises simultaneously with intense epithermal microwave radiation at a frequency close to /131

$$\omega = \sqrt{\omega_0^2 + \omega_H^2} \cdot \sqrt{\frac{M_e}{M_i}}$$

(ω_0 -- plasma electron frequency; ω_H -- electron cyclotron frequency). The power of this radiation is approximately four orders of magnitude greater than the power of thermal radiation from the plasma at an electron temperature of 10^4 electronvolts. It was shown in (Ref. 10) that the radiation is related to longitudinal electron oscillations in the discharge.

All of these statements indicate that X-ray radiation from discharge is related to the thermalization of bunched instability energy. /132

The period of pair interaction for electrons having an energy on the order of 2 kev is considerably greater than the time of the process being studied. Therefore, we may assume that nonlinear phenomena during collective oscillations with a large amplitude play the main role in thermalization of longitudinal oscillations. In our case, the Larmor radius of hot electrons is a little less than the discharge chamber diameter (less than 1 mm). Therefore, X-ray radiation is primarily caused by electrons diffusing toward the walls across the magnetic field. When the X-ray radiation reaches a maximum (see Figure 4), the luminescence in the discharge center sharply increases; this is determined primarily by admixtures dislodged from the walls by hot electrons. There is a simultaneous strong increase in the current on the boundary electrostatic probe, which is executed in such a way that the current upon it is determined by the resistance of the discharge plasma across the magnetic field. Thus, there is a rapid cooling of the electrons, even 5 - 6 microseconds after the X-ray radiation has terminated.

In the absence of pair collisions, strong diffusion and great conductivity across the magnetic field would not be expected. Anomalous diffusion can occur due to nonlinear phenomena for a large longitudinal oscillation amplitude. However, in the experiments described, the anomalous diffusion across the magnetic field may be explained by an increase in the gaskinetic plasma pressure as compared with the magnetic pressure, due to intense electron heating. The decrease in the X-ray radiation intensity with an increase in the magnetic field strength also points to this conclusion. /133

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DAMPING OF INITIAL PERTURBATION AND STEADY FLUCTUATIONS IN A COLLISIONLESS PLASMA

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When the perturbation of the distribution function is a nonanalytical function of velocity, the damping of plasma waves when there are no collisions may be different from that determined by the classical formula of Landau. If the perturbation of the distribution function has a δ-like singularity, then perturbations of the macroscopic quantities are not damped with the passage of time. If the perturbation of the distribution function has a discontinuity of the n-th derivative, then the perturbation is damped according to the law $t^{-(n+1)}$ (and not according to the exponential law, as in the case of Landau damping). /134

Let us investigate the mechanism for establishing fluctuations (which are not dependent on the initial perturbations) of the macroscopic quantities in a non-equilibrium plasma. We shall show that these fluctuations are established due to the "survival" of a singular component in the expression for the distribution function perturbation.

The dependence on time of the perturbation of the k-th component of the Fourier potential $\phi_k(t)$ in an unconfined plasma is determined by the following expression (Ref. 1)

$$\varphi_k(t) = (2\pi i)^{-1} \int\limits_{\sigma - i\infty}^{\sigma + i\infty} \varphi_{kp} e^{pt} dp, \tag{1}$$

where

$$\varphi_{kp} = -\frac{4\pi e^2}{k^2} \cdot \frac{N(\mathrm{k}, p)}{D(\mathrm{k}, p)}, \tag{2}$$

$$D(\mathrm{k}, p) = 1 - \frac{4\pi i e^2}{mk} \int\limits_{-\infty}^{\infty} F_0'(w) \frac{dw}{p + ikw}; \tag{3}$$

$$N(\mathrm{k}, p) = \int\limits_{-\infty}^{\infty} g_k(w) \frac{dw}{p + ikw}; \tag{4}$$

$$w = \frac{\mathrm{kv}}{k}, \; F_0(w) = \int F_0(\mathrm{v})\, d\mathrm{v}_\perp, \; g_k(w) = \int g_k(\mathrm{v}) d\mathrm{v}_\perp, \; \mathrm{v}_\perp = \mathrm{v} - \mathrm{k}\frac{w}{k};$$

$F_0(v)$ is the unperturbed distribution function; $g_h(v)$ -- the Fourier component of the distribution function perturbation in the case of t = 0; integration is performed in formula (1) along the line Re p = σ lying to the right of all the singularities of the function ϕ_p.

Formula (1) makes it possible to determine the behavior of the potential $\phi_k(t)$ with an increase in t. As is known, the asymptotic behavior of the function $\phi_k(t)$ for large t is determined by the nature of the singularities of the Laplace transform ϕ_p. The function ϕ_p was determined above for only large values of Re p (in this region it has no singularities). In order to study its singularities, we must first determine this function in the entire complex variable plane -- i.e., we must analytically continue the determination of (2) to decreasing values of Re p. The analytical continuation of ϕ_p is determined according to the previous formula (2) along the imaginary axis p.

For purely imaginary values of p, the denominators in the integrals /135 which determine ϕ_{cr} vanish in the case of $w = ip/k$. Therefore, for analytical continuation of ϕ_{cr} in the region Re $p \leqslant 0$, it is necessary to deform the integration path in the integrals (3) and (4), so that it passes around the pole $w = ip/k$ from below. Deformation of the path assumes, in its turn, the possibility of analytical continuation of the functions $F_0'(w)$ and $g_k(w)$ determined initially only for real w in the region of complex values of w.

Thus, a clarification of the singularitites for the function ϕ_{cr}, which determines the nature of the asymptotic behavior of $\phi_k(t)$ for large t, requires a knowledge of the analytical properties of the functions $F_0(w)$ and $g_k(w)$.

Let us confine ourselves to investigating the functions $F_0(w)$ permitting analytical continuation in the region of complex values. The function D(k, p), determined in the case of Re p > 0 by relationship (3), may be continued analytically in the region Re $p \leqslant 0$, by determining it everywhere as

$$D(k, p) = 1 - \frac{4\pi i e^2}{mk} \int\limits_C \frac{F_0'(w)\,dw}{p + ikw}, \tag{5}$$

where integration is performed along the real axis w with passage around the pole from below in the case of $w = ip/k$.

We have found the denominator of expression (2) for ϕ_p -- i.e., the function D(k, p) -- over the entire plane of the complex variable p. Let us now calculate the analytical properties of the numerator for this expression, i.e., the function N(k, p). Formula (4) determines it in the case of Re p > 0. As has been pointed out, the function N(k, p) has no singularities in this region. The position and nature of the singularities of this function are determined by the properties of the function $g_k(w)$ in the case of Re $p \leqslant 0$.

If the function $g_k(w)$ has singularities (which may be integrated) for real w, then the function N(k, p) will have singularities for purely imaginary p. In particular, such a situation is observed if the function $g_k(w)$ has a δ-like singularity, a discontinuity, or a break, and also if any of its derivatives has a break (in these cases, the function $g_k(w)$, generally speaking, does not permit analytical continuation on the real axis.

If the function $g_k(w)$ has no singularities on the real axis and permits analytical continuation in the region of complex values of w, then the function N(k, p), and consequently the function ϕ_p, will have no singularities on the imaginary axis p. However, generally speaking, it may have a singularity in the case of Re p < 0 at the points $p = -ikw_r$, where w_r is any singularity of the function $g_k(w)$ lying in the lower halfplane of the complex variable w. /136

Let us elaborate further on the nature of the asymptotic behavior of $\phi_k(t)$ for noninteger functions $g_k(w)$. In this case, singularities of the function N(k, p) are added to the singularity ϕ_p determining the roots of the dispersion equation D(k, p) = 0. The distribution of these singularities depends only on the form of the function $g_k(w)$ -- i.e., on the nature of the initial perturbation -- and does not depend on the plasma properties (on the function $F_0(w)$.) As has been indicated, one significant property of the singularities for the function of N is the fact that they may all lie only in the left halfplane p. Therefore, if only one of the roots $p_r = -i\omega_r - \gamma_r$ of the dispersion equation D(k, p) = 0 lies in the right halfplane p, $\gamma_r < 0$ (which corresponds to the possibility of an oscillation increase), then the nature of the initial perturbation has no significant influence on the asymptotic behavior of $\phi_k(t)$ in the case t $t \to \infty$.

If N(k, p) has singularities at the points $p = p_n \equiv -\gamma_n - i\omega_n$ (n = 1, 2, 3, ...), the contribution made by these singularities to the asymptotic behavior of $\phi_k(t)$ in the case of $t \to \infty$ may be written as $\sum_n \alpha_n \exp\{-\gamma_n t - i\omega_n t\}$,

where α_n represents certain constants. Adding this sum to the contribution from zeros D(k, p), we find the asymptotic expression for $\phi_k(t)$ in the general case of noninteger functions $g_k(w)$ (which have no singularities for real w)

$$\varphi_k(t) \sim \sum_r \varphi_{kp}^{(r)} \exp\{-\gamma_r t - i\omega_r t\} + \sum_n \alpha_n \exp\{-\gamma_n t - j\omega_n t\}, \tag{6}$$

where $\phi^{(r)}_{cr}$ is the residue ϕ_{cr} at zero of the function D(k, p) (the point $p = p_r \equiv -\gamma_r - i\omega_r$).

Thus, for large t the potential $\phi_k(t)$ represents superposition of the eigen plasma oscillations, whose complex frequecies $\omega_r - i\gamma_r$ are determined by the plasma properties [the right sum in (6)], and oscillations whose complex frequencies $\omega_n - i\gamma_n$ are determined by the form of the initial perturbation $g_k(w)$ [the second sum in (6)]. The eigen oscillations may be both damped and intensified. The oscillations whose frequencies are determined by the form of the function $g_k(w)$ may be only nonincreasing (i.e., damped or oscillating oscillations with constant amplitude).

Let us give two examples of oscillations whose frequency and damping decrement are determined by the initial perturbation, and do not depend on the plasma properties.

As the first example, let us investigate oscillations produced in the case of /137

$$g_k(w) = \frac{g_0 w_1}{(w - w_0)^2 + w_1^2}, \tag{7}$$

where g_0, w_0, w_1 are certain constants. In this case we have

$$N(\mathrm{k},\ p) = -\frac{\pi g_0}{p + ikw_0 + kw_1}.$$

The function $N(k, p)$ has a singularity in the case of $p = -ikw_0 - kw_1$, which introduces the following contribution to the asymptotic behavior of $\phi_k(t)$ in the case of $t \to \infty$

$$\varphi_k(t) \sim g_0 \exp\{-kw_1 t - ikw_0 t\}. \tag{8}$$

Thus, the frequency and damping decrement of oscillations produced in the case of initial perturbation such as (7) equal kw_0 and $kw_\perp$, respectively. In the case of $w_1 \to 0$, the damping disappears. We should point out that the function $g_k(w)$ acquires an δ-like singularity on the real axis, $g_k(w) \to \pi g_0 \delta(w - w_0)$.

Let us study oscillations produced in the case of the discontinuous functions $g_k(w)$. Let us set, for example,

$$g_k(w) = \begin{cases} g_0 & (-w_0 < w < w_0), \\ 0 & (|w| > w_0). \end{cases} \tag{9}$$

Thus, the function

$$N(\mathbf{k},\ p) = \frac{g_0}{ik} \ln \frac{p + ikw}{p - ikw}$$

has branch points on the imaginary axis p, $p = \pm ikw$. The contribution made by the singularities of the function $N(\mathbf{k}, p)$ to the perturbation of the potential $\phi_k(t)$ has the following form

$$\varphi_k(t) \sim g_0 \frac{\sin(kw_0 t)}{t}. \qquad (10)$$

The δ-like singularity on the real axis on the function $g_k(w)$ leads to nondamped oscillations of the potential $\phi_k(t)$. The discontinuity of the function $g_k(w)$ -- i.e., the δ-like singularity of its first derivative -- leads to potential oscillations which are damped as t^{-1}. It can readily be shown that the discontinuity of the n-th derivative of the function $g_k(w)$ -- i.e., the δ-like singularity of its (n + 1)-th derivative -- leads to asymptotic behavior of a $t^{-(n+1)}\exp\{ikw_0 t\}$ like potential, where w_0 is the discontinuity point.

Let us determine the manner in which the fluctuations of macroscopic quantities, which do not depend on the initial conditions, are established in a collisionless plasma with an arbitrary (not necessarily equilibrium) distribution function (Ref. 2). In order to do this, we should note that the averaged product of the distribution function fluctuations for particles of the a-th type can be represented as follows at corresponding periods of time /138

$$\langle g_k^a(\mathbf{v})\, g_{k'}^{a'}(\mathbf{v}') \rangle = \delta_{aa'}\delta(\mathbf{k} + \mathbf{k}')\delta(\mathbf{v} - \mathbf{v}')F_0^a(\mathbf{v}) + Y_{aa'}(\mathbf{v}, \mathbf{v}'; \mathbf{k}, \mathbf{k}'), \qquad (11)$$

where the first component (Ref. 3) describes the "correlation of the particle with itself", and the second component is related to the interaction between particles (and is determined by the previous history of the system). It is important that the first component contain $\delta(\mathbf{v} - \mathbf{v}')$, while the second component is a smooth function of velocity.

In order to obtain the correlation function of the potential, we must express the potential $\phi_k(t)$ by $g_k^a(\mathbf{v})$ by means of relationships (1) - (4), and we must then perform averaging by means of formula (11). It may be readily seen that the presence of the δ-like component in formula (11) leads to a nondamped (and oscillating according to the law $\exp\{i\mathbf{k}\mathbf{v}t\}$) component in the expression for the potential correlator. The remaining (smooth) components in formula (11) are damped according to the law $\exp\{-\gamma_k t\}$, where γ_k is the customary Landau damping decrement (i.e., the imaginary part of the root ω_k of the dispersion equation $\varepsilon(\omega_k, \mathbf{k}) = 0$).

Thus, in the time γ_k^{-1} the potential fluctuation distribution (and the distribution of all other macroscopic quantities), which does not depend on the initial perturbation, is established after the outer perturbation is shut off in the plasma.

According to relationships (1) - (4), (11), the correlation function of the potential has the following form

$$\langle \varphi_k(t)\varphi_{k'}(0)\rangle = \delta(\mathbf{k}+\mathbf{k}')\left(\frac{4\pi}{k^2}\right)^2 \int_{-\infty}^{\infty} d\omega\,|\varepsilon(\omega,\mathbf{k})|^{-2}\exp\{-i\omega t\}\times \times \sum_a e_a^2 \int d\mathbf{v} F_0^a(\mathbf{v})\,\delta(\omega-\mathbf{k}\mathbf{v}). \quad (12)$$

This relationship was obtained by Rostoker (Ref. 4) by employing a different method. Our derivation presents a clearer explanation of the mechanism, and makes it possible to determine the time required to establish fluctuations, which do not depend on the initial perturbation, in a nonequilibrium plasma.

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CHARGED PARTICLE INTERACTION WITH A TURBULENT PLASMA

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This article computes the energy lost (or acquired) per unit of time by a charged particle when moving through a turbulent plasma. The dependence of the particle energy change on the magnitude and direction of its velocity is established in explicit form. It is shown that the turbulence spectrum elements have no influence on this dependence.

The strength of charged particle interaction with the plasma is determined by the level of plasma fluctuations. In particular, the particle energy losses per unit of time P are related to the charge density correlator $\langle\rho^2\rangle_{q\omega}$ by the following relationship

$$P = -\frac{(ez)^2}{\hbar}\int\left(\frac{4\pi}{q^2}\right)^2 \omega \langle \rho^2 \rangle_{q\omega}\delta\left(\omega - \mathbf{qv} - \frac{\hbar q^2}{2\mu}\right)\frac{d\omega dq}{(2\pi)^3}, \tag{1}$$

where ez, μ, **v** are, respectively, the particle charge, mass, and velocity. If the plasma consists of cold ions and hot electrons moving at the mean velocity **u** with respect to the ions, the charge density correlator in the "sound region" $(q(T_i/M)^{1/2} \ll \omega \ll q(T_e/m)^{1/2},\ aq \ll 1)$ may be represented in the following form

$$\langle \rho^2 \rangle_{q\omega} = \frac{1}{4} q^2 (aq)^2 \left\{\left[T(q,\ \mathbf{qu}) - \frac{1}{2}\hbar\omega\right]\delta(\omega - qs) + \right.$$
$$\left. + \left[T(q,\ -\mathbf{qu}) - \frac{1}{2}\hbar\omega\right]\delta(\omega + qs)\right\}, \tag{2}$$

where T_e, T_i is the temperature and m, M -- the mass, respectively, of electrons and ions; $s = (T_e/M)^{1/2}$ -- speed of sound; T(q, qu) -- effective temperature of sound waves; $a = T_e^{1/2}(4\pi e^2 n)^{-1/2}$ -- Debye radius. Substi- /140 tuting (2) in (1) and assuming that $T \gg \mu v^2$, we obtain

$$P = \frac{(eza)^2 us}{\pi\mu v^2}\int q^4 dq \int_0^\pi d\varphi \left\{\cos\theta - \left(\frac{v^2}{s^2} - 1\right)^{-1/2}\sin\theta\cos\varphi\right\}\frac{\partial}{\partial\eta}T(q,\eta), \tag{3}$$

where θ is the angle between **v** and **u**

$$\eta \equiv \mathbf{qu} = qu\left\{\frac{s}{v}\cos\theta + \left(1 - \frac{s^2}{v^2}\right)^{1/2}\sin\theta\cos\varphi\right\}.$$

In the wave vector region in which the sound waves are damped (qu < < qs), the effective temperature is (Ref. 1 - 3) $T_e(1 - qu/qs)^{-1}$. Close to the boundary of the stability region (qu $\stackrel{\sim}{<}$ qs) the function T increases sharply. In the case of qu > qs, the linear theory predicts an exponential increase in T with time. This increase stops due to nonlinear phenomena, and a stationary fluctuation distribution is established which is characterized by a very high effective temperature, i.e., the state of stationary turbulence (Ref. 4).

It follows from this behavior of the function $T(q,\eta)$ that $\partial T(q,\eta)/\partial\eta$ has a sharp maximum for a certain value of η close to qs, $\eta = \eta_0 \approx qs$. Noting that the energy losses are determined by the derivative $\partial T/\partial\eta$, and not by the function T itself, we can thus express P by a small number of parameters which characterize $\partial T/\partial\eta$ in the case of $\eta \approx \eta_0$, without including more detailed properties of the turbulence spectrum.

Close to $\eta \approx \eta_0$, we have

$$\left\{\frac{\partial}{\partial\eta}T(q,\eta)\right\}^{-1} = \frac{qs}{T_e}\left\{\left(\frac{T_e}{T^*}\right)^2\lambda_1^2(q) + \lambda_2^2(q)\left(1-\frac{\eta}{\eta_0}\right)^2\right\}, \tag{4}$$

where $\lambda_{1,2}$ equals unity in order of magnitude, and T* is a large quantity equalling $T(q, \eta)$ in order of magnitude in the case of $\eta > qs$.

Substituting (4) in formula (3) and assuming that $\theta_+ < \theta < \theta_-$, where

$$\cos\theta_\pm = (uv)^{-1}\{s^2 \pm (v^2 - s^2)^{1/2}(u^2 - s^2)^{1/2}\}, \tag{5}$$

we obtain

$$P = \frac{(ez)^2T^*s}{a^2\mu uv}\alpha\frac{uv - s^2}{v^2 - s^2}(\cos\theta_+ - \cos\theta)^{-1/2}(\cos\theta - \cos\theta_-)^{-1/2}, \tag{6}$$

where $\alpha = a\int(aq)^3(\lambda_1\lambda_2)^{-1}dq$. This relationship determines the dependence of P on v in explicit form. In the case of $\theta < \theta_0$ (θ_0 = arc cos s^2/uv), the particle energy decreases and in the case of $\theta > \theta_0$ it increases.

In the case of $|\theta - \theta_\pm| \lesssim T_e/T^*$, relationship (6) ceases to be valid. /141
In the case of $|\theta - \theta_\pm| \ll T_e/T^*$, we have

$$P = \pm\frac{(ez)^2(T^*)^{3/2}s^{1/2}}{a^2\mu(uvT_e\sin\theta_\pm)^{1/2}}\alpha_1(u^2 - s^2)^{1/2}(v^2 - s^2)^{-3/4}, \tag{7}$$

where $\alpha_1 = \frac{1}{2}a\int(aq)^3\lambda_1^{-3/2}\lambda_2^{-1/2}dq$ (the signs «$\pm$» correspond to $\theta \approx \theta_\pm$). Thus, P is proportional to T* in the region $\theta_+ < \theta < \theta_-$, except for the boundary of this region where $P \sim (T^*)^{3/2}$.

In the case of $v \to u$, the critical value of the angle θ_+ strives to zero, and relationship (7) does not hold. For $\theta_+ \ll T_e/T^*$, and employing formulas (3), (4), we obtain

$$P = \frac{(ezT^*)^2}{a^2\mu v T_e}\alpha_2, \quad \alpha_2 = a\int(aq)^3\lambda_1^{-2}\,dq. \qquad (8)$$

In this case, the energy losses are particularly large (they are proportional to (T^{*2}).

If $\theta < \theta_+$ or $\theta > \theta_-$, then the expression for the energy loss does not contain the large parameter T^*/T_e. For $|\theta - \theta_\pm| \ll 1$, nevertheless, P is proportional to $|\theta - \theta_\pm|^{-3/2}$, and consequently it is large.

In order to have critical values of the angles $\theta_\pm$, it is necessary that both u and v exceed s. If $u \to s$, then $\theta_\pm \to \theta_c = \arccos s/v$. In the case of $\theta \approx \theta_c$, the energy losses are propotional to $(T^*)^{1/2}$

$$P = \frac{(ezs)^2(T^*T_e)^{1/2}}{a^2\mu(v^2 - s^2)}\alpha_0, \quad \alpha_0 = \frac{1}{2}a\int(aq)^3\lambda_1^{-1/2}\lambda_2^{-3/2}dq. \qquad (9)$$

When expressions (6) - (9) were derived, it was assumed that the difference v - s was not too small. If $1 - s^2/v^2 \ll T_e/T^*$, the energy losses will be at a maximum in the case of $\mathbf{vu} = s^2$ (in this case P is determined by formula (8) and sharply decreases with an increase in $|\mathbf{vu} - s^2|$.

In conclusion, we would like to point out that the dependence of P on the angle θ holds, even if the assumption that $\partial T/\partial\eta$ is small in the region $\eta > \eta_0$ is not fulfilled. The contribution made by the quantity $\partial T/\partial\eta$ with $\eta > \eta_0$ in the expression for P can only change the function P somewhat in the case of $\theta_+ < \theta < \theta_-$, without changing P in the case of $\theta \approx \theta_\pm$. Consequently, the nature of the dependence of P on the angles is not changed in the case of $\theta_+ \lesssim \theta \lesssim \theta_-$. In particular, the function P, which is positive in the case of $\theta = \theta_+$ and negative in the case of $\theta = \theta_-$, must vanish for a certain value of the angle $\theta = \theta_0$, $\theta_+ < \theta_0 < \theta_-$ (thus θ_0 can differ somewhat from $\arccos s^2/uv$).

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/142

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THEORY OF NONLINEAR MOTIONS OF A NONEQUILIBRIUM PLASMA

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As is well known, low frequency oscillations with a linear law of dispersion -- so-called ion sound -- are possible in a collisionless plasma consisting of hot electrons and cold ions (Ref. 1, 2). It is interesting to study nonlinear motions of a collisionless plasma consisting of hot electrons and cold ions, and primarily simple waves. The study of simple waves not only makes it possible to trace the development of an initial perturbation, but it is also of interest as an independent investigation, since only the region of simple waves can (when there are no discontinuities) be contiguous to an unperturbed plasma [see (Ref. 3)].

A. A. Vedenov, Ye. P. Belikhov, and R. Z. Sagdeyev (Ref. 4) have studied simple waves in a two-temperature plasma on the basis of an iosthermal hydrodynamic model. This article investigates simple waves in a nonequilibrium plasma on the basis of a kinetic equation, without employing a special model. A system of equations has been obtained for the moments (introduced in a specific way) of the electron distribution function, which enabled us to determine the direction of change for quantities characterizing the plasma in a sound wave, and to trace the development of a perturbation having finite amplitude*.

Equations Describing a Simple Wave /143

The system of equations describing the motion of a collisionless

* Yu. L. Klimontovich and V. P. Silin (Ref. 5) have investigated the problem of a hydrodynamic description of a two-temperature plasma without collisions. Nonlinear motions of such a plasma were studied in (Ref. 4, 6).

plasma consisting of hot electrons and cold ions has the following form

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{e}{m}\mathbf{E}\frac{\partial}{\partial \mathbf{v}}\right)F=0;$$
$$\left(\frac{\partial}{\partial t}+\mathbf{u}\frac{\partial}{\partial \mathbf{r}}\right)\mathbf{u}+\frac{e}{M}\mathbf{E}=0;\ \frac{\partial}{\partial t}n+\operatorname{div}(n\mathbf{u})=0; \tag{1}$$
$$\operatorname{div}\mathbf{E}=4\pi e\left(\int F d\mathbf{v}-n\right);\quad \operatorname{rot}\mathbf{E}=0,$$

where $F(\mathbf{v})$ is the electron distribution function; n and $\mathbf{u}$ are, respectively, the ion density and hydrodynamic velocity; $\mathbf{E}$ -- electric field; m, M -- electron and ion masses, respectively. (It is assumed that the electron mean energy considerably exceeds the ion mean energy.) Being interested in sound oscillations whose phase velocity is small as compared with the mean thermal velocity of electrons, we do not have to take the term $\partial F/\partial t$ into account in the first of the equations (1). Confining ourselves to one-dimensional plasma motions and making allowance for the fact that the charge spatial distribution is small in a soundwave

$$\left|n-\int F d\mathbf{v}\right|\sim n(a/\lambda)^2\ll n$$

(λ -- the length at which the quantities characterizing the plasma change significantly; a -- Debye radius), we can reduce the system of equations (1) to the following form

$$\frac{\partial F}{\partial x}-\frac{M}{m}\cdot\frac{du}{dt}\cdot\frac{\partial F}{v_x\partial v_x}=0;\quad \frac{dn}{dt}+n\frac{\partial u}{\partial x}=0; \tag{2}$$
$$n=\int F d\mathbf{v},$$

where $d/dt = \partial/\partial t + u\partial/\partial x$ (the x axis is selected in the direction of wave propagation; the subscript x for the velocity component u_x is omitted from this point on).

Let us introduce the "moments of the distribution function":

$$D_j(x,t)=(-2)^j\int\frac{\partial^j}{\partial\left(v_x^2\right)^j}F(\mathbf{v};x,t)\,d\mathbf{v},\quad j=0,1,\ldots. \tag{3}$$

Employing system (2), we obtain

$$\frac{\partial D_j}{\partial x}+\frac{M}{m}\cdot\frac{du}{dt}D_{j+1}=0;\quad \frac{dn}{dt}+n\frac{\partial u}{\partial x}=0;\quad D_0=n. \tag{4}$$

In order to study nonlinear plasma motions, system (4) is more advantageous than the initial system of equations (2), since it includes terms which are only dependent on x and t, while equations (2) also include the electron velocity distributions. In this sense, equations (4) are similar /144 to equations of hydrodynamics, although -- in contrast to hydrodynamics which operate with a finite number of quantities -- they include an infinite number of "hydrodynamic quantities" n, D_j, u.

We are interested in simple waves, i.e., those plasma motions for which the perturbations of all quantities characterizing the plasma are propagated at the same velocity -- in other words, for which each of the functions X [$X \equiv u, n, D_j, F(\mathbf{v})$] satisfies the following equation

$$\left(\frac{\partial}{\partial t} + V(x, t)\frac{\partial}{\partial x}\right) X = 0.$$

In the case of simple waves, as is well known, all of the quantities X can be represented in the form of a function of one of them (for example, n), which in its turn is a function of x, t. System (4) thus changes into a system of customary differential equations for the functions $D_j(n)$, u(n), and the phase velocity V(n) is determined from the solvability condition of this system. After simple transformations, we obtain

$$\begin{gathered} \frac{du}{dn} = \varepsilon\frac{V_s}{n}; \quad \frac{dD_j}{dn} = \frac{D_{j+1}}{D_1}; \\ V = u + \varepsilon V_s, \quad V_s = \left(\frac{mn}{MD_1}\right)^{1/2}, \end{gathered} \tag{5}$$

where $\varepsilon = +1$ ($\varepsilon = -1$), if the wave is propagated in the positive (negative) direction of the x axis.

The determination of the electron distribution function in the case of a simple wave may also be reduced to solving the customary differential equation. Rewriting the kinetic equation (2) in the following form

$$\frac{\partial F(\mathbf{v})}{\partial n} + \frac{M}{m}\cdot\frac{(V-u)^2}{n}\cdot\frac{\partial F(\mathbf{v})}{v_x \partial v_x} = 0$$

and introducing the notation

$$F\left(v_x^2;\ \mathbf{v}_t;\ n\right) \equiv F(\mathbf{v};\ n)$$

[$\mathbf{v}_t \equiv (v_y, v_z)$], we obtain

$$F\left(v_x^2;\ \mathbf{v}_t;\ n\right) = F\left(v_x^2 + \beta(n);\ \mathbf{v}_t\right), \tag{6}$$

where the function β(n) satisfies equation /145

$$\frac{d\beta}{dn} + \frac{2M}{m}\cdot\frac{V_s^2}{n} = 0. \tag{7}$$

We should point out that Landau damping of sound waves was not taken into account when the initial equations (2) were derived. Therefore, it is necessary that the wave amplitude Δn not be too small, $\Delta n/n \gg (m/M)^{1/2}$, in order that equations (2), and consequently relationships (4) - (7) may be valid. The role of nonlinear phenomena in the development of the perturbation is much greater in fulfilling this condition, than is the role of sound damping.

Development of a Perturbation Having a Finite Amplitude

The system of equations for "hydrodynamic" quantities (5), together with relationships (6) and (7), enables us to study the direction of the change in quantities characterizing the plasma (including the electron distribution function) and to trace the development of a perturbation having a finite amplitude.

First of all, let us determine the manner in which the electron distribution function changes in a simple wave. It follows from (7) that β decreases in a contraction wave, and increases in a rarefaction wave. Therefore, for values of $\mathbf{v}$ for which $\partial F/v_x\partial v_x < 0$, the number of electrons having a velocity in the $(\mathbf{v}, \mathbf{v} + d\mathbf{v})$ range increases in the contraction wave, and decreases in the rarefaction wave. Conversely, at values of $\mathbf{v}$ for which $\partial F/v_x\partial v_x > 0$, the number of electrons with velocities in the $(\mathbf{v}, \mathbf{v} + d\mathbf{v})$ range increases in a rarefaction wave, and decreases in a contraction wave. In particular, if the initial electron velocity distribution has a spike encompassing a small velocity region, along with a maximum for $v_x = 0$, the spike shifts to the region of larger (smaller) values of $|v_x|$ as the contraction wave (rarefaction) moves.

Let us dwell in somewhat greater detail on the case of Maxwell distribution. Employing system (5), we may state that in this case V_s, D_j/n do not depend on n, and consequently are motion integrals. The electron temperature and the distribution function F/n, which is normalized to one particle, do not change during wave propagation. Thus, in the case of a Maxwell velocity distribution of electrons it is valid to describe a two-temperature plasma by means of isothermic hydrodynamics.

In order to determine the manner in which the form of the sound wave changes, it is necessary to compute the derivative dV/dn [see (Ref. 7)]. Employing the system of equations (5), and assuming, for purposes of definition, that $\varepsilon = 1$, we obtain /146

$$\frac{dV}{dn} = \frac{V_s}{2n}\left(3 - \frac{nD_2}{D_1^2}\right). \quad (8)$$

Depending on the electron distribution function, dV/dn may be positive, negative, equal to zero, or alternating [positive for single values of the parameter β and negative for other values of this parameter, see formula (6)]. (We should point out that the derivative dV/dn is always positive both in customary and in magnetic hydrodynamics.)

If dV/dn > 0 for all values of the parameter β, then (just as in customary hydrodynamics) points with a large density move at a large velocity. Therefore, discontinuities arise at the contraction sections.** Self-similar waves are rarefaction waves. In particular, this possibility exists for a Maxwell electron velocity distribution and for a distribution in the form of a step $F \sim \Theta\ (v_0^2(n) - v^2)$, $\Theta\ (x) = \frac{1}{2}\ (1 + \mathrm{sign}\ x)$.

If for all values of β, dV/dn = 0, all the points move at the same velocity during wave propagation. Therefore, the wave profile is not deformed and no discontinuities arise. Employing equations (5) and (7), we may state that the velocity of two-temperature sound and the quantity $\beta^{1/2}$ change in an inverse proportion to density, $V_s n = \mathrm{const}$, $\beta n^2 = \mathrm{const}$. The case dV/dn = 0 is realized, in particular, for a Cauchy distribution $F \sim \{v_0^2(n) + v^2\}^{-2}$.

If dV/dn < 0 (independently of the value for the parameter β), then points with a large density move at a low velocity. Therefore, discontinuities arise in the rarefaction sections. Self-similar waves are contractions waves. This possibility is realized, in particular, if the distribution function is the superposition of two Cauchy distributions

$$F \sim \nu_1(n)\{v^2 + v_1^2(n)\}^{-2} + \nu_2(n)\{v^2 + v_2^2(n)\}^{-2}.$$

We should note that in this case the velocity V_s increases in the rarefac- /147
tion wave, and decreases in the contraction wave.

Finally, let us discuss the case when dV/dn may be both positive and negative, depending on the value of the parameter β. For purposes of definition, we shall assume that dV/dn > 0 in the case of $\beta > \beta_1$ and dV/dn < 0 in the case of $\beta < \beta_1$, where β_1 is a certain critical value of the parameter β. When a contraction wave moves in such a plasma, the

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** We have employed the term "discontinuity" to designate the narrow regions in which the gradients of the quantities characterizing the plasma become so large that the initial equations (2) are not applicable. In the case of $a/\lambda \gtrsim 1$, sound dispersion must be taken into account. With a further increase in the gradients, multi-flux flows [see (Ref. 4)] or shockwaves may arise in these regions.

"apex" of the wave (points with the density $n > n_1$, where n_1 is determined from the equation $\beta(n_1) = \beta_1$) lags behind the "base" of the wave (points with $n < n_1$). Therefore, the density at the point of the discontinuity which develops from such a wave cannot exceed n_1. A discontinuity may also arise during the motion of a rarefaction wave. Thus, the density cannot be less than n_1 at the discontinuity point. If, on the other hand, $dV/dn < 0$ in the case of $\beta > \beta_2$ and $dV/dn > 0$ in the case of $\beta < \beta_2$, then -- as may be readily confirmed -- the density at the discontinuity point cannot exceed n_2 when a discontinuity develops from a rarefaction wave, and cannot be less than n_2 when a discontinuity develops from a contraction wave (the critical density n_2 is determined from the equation $\beta(n_2) = \beta_2$).

Both of the above possibilities may be realized, in particular, if the electron velocity distribution is a superposition of two Maxwell distributions, "hot" and "cold",

$$F(v;\, n) = \nu_1(n)\exp\left\{-\frac{mv^2}{2T_1}\right\} + \nu_2(n)\exp\left\{-\frac{mv^2}{2T_2}\right\},\ T_1 \gg T_2.$$

For small density values $\left(n \ll (\alpha_1 T_1^{1/2}/\alpha_2 T_2^{1/2})^{T_2/T_1}\right)$, according to equations (6), (7), $\nu_1 = \alpha_1 n;\ \nu_2 = \alpha_2 n^{T_1/T_2}$ (α_1, α_2 -- constants). It may be readily confirmed that in this case $dV/dn > 0$ in the case of $n < n_1$ and $dV/dn < 0$ in the case of $n > n_1$, where $n_1 = \left(\frac{2\alpha_1}{\alpha_2}\cdot\frac{T_2^{1/2}}{T_1^{1/2}}\right)^{T_2/T_1}$. For large density values $\left(n \gg \alpha_1' T_1^{1/2}/\alpha_2' T_2^{1/2}\right)$ $\nu_1 = \alpha_1' n^{T_2/T_1};\ \nu_2 = \alpha_2' n$ (α_1', α_2' -- constants). In this case, $dV/dn < 0$ for $n < n_2$ and $dV/dn > 0$ in the case of $n > n_2$, where $n_2 = \frac{\alpha_1}{2\alpha_2'}(T_1/T_2)^{3/2}$.

Let us investigate the motion of a two-temperature plasma arising during its uniform contraction or expansion [similarly to the problem of the plunger in hydrodynamics, see the monograph (Ref. 7)]. We shall assume that the plasma occupies the halfspace $x > V_0 t$, which is uniformly /148
limited by a moving plane. (Such a boundary may represent, in particular, the region of a very strong magnetic field.) As is well known, only self-similar waves (in the absence of shock waves) can be steady motions of a uniformly contracted (or expanding) medium. If $dV/dn > 0$, a self-similar wave (which is in this case a rarefaction wave) arises during plasma expansion ($V_0 < 0$). If $dV/dn < 0$, a self-similar wave (which is in this case a contraction wave) arises during plasma contraction ($V_0 > 0$).

Employing formulas (5), (6), (7), we may relate the change in all the quantities X characterizing the plasma in a self-similar wave with the "plunger" velocity V_0. Assuming, for purposes of simplicity, that

$V_0 \ll V_s$, we obtain

$$\begin{aligned}
\Delta n &= n \frac{V_0}{V_s}; \quad \Delta D_i = n \frac{D_{i+1}}{D_1} \cdot \frac{V_0}{V_s}; \\
\Delta V_s &= \frac{1}{2}\left(1 - \frac{nD_2}{D_1^2}\right) V_0; \\
\Delta F(\mathrm{v}) &= -\frac{M}{m} V_s V_0 \frac{\partial F(\mathrm{v})}{v_x \partial v_x}; \\
\Delta X &= X|_{x=V_s t} - X|_{x\to\infty}.
\end{aligned} \tag{9}$$

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NONLINEAR PROCESSES IN A UNIFORM AND ONE-COMPONENT PLASMA

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Nonlinear solutions of a one-dimensional kinetic equation without a collision term, which depend on x and t by the combination $\xi = x - V_0 t$, where -- V_0 is the constant wave velocity, were compiled in (Ref. 1) and studied for several cases in (Ref. 1 - 4). The conditions at which these solutions may be realized were found in (Ref. 4), and a limiting transition to small oscillations was performed. /149

General Theory of Nonlinear Waves

This article investigates the more general nonlinear solutions of a one-dimensional kinetic equation without collisions

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} - \frac{\partial \varphi}{\partial x}\cdot\frac{\partial f}{\partial u} = 0, \tag{1}$$

where ϕ is the potential of the self-consistent electric field with the factor $\frac{e}{m}$, determined by the Poisson equation

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{4\pi e^2}{m}\left(\int f du - n_0\right), \tag{2}$$

where n_0 is the unperturbed density of ions whose mass is assumed to be infinitely large. It is assumed that the distribution function f depends only on the variables u and ϕ. Then the electron density

$$\langle n \rangle = \int f du = F(\varphi), \tag{3}$$

the density of the electron flux

$$\langle nu \rangle = \int uf du = \Phi(\varphi) \tag{4}$$

and the energy density

$$\left\langle n\frac{mu^2}{2} \right\rangle = \int f(u)\frac{mu^2}{2} du = \Psi(\varphi) \tag{5}$$

are explicit functions of only the potential ϕ.

We shall show that in this case the main plasma characteristics may be compiled within the framework of a hydrodynamic approximation, and that the electron velocity distribution function may be found relatively simply. From the equation of continuity

$$\frac{\partial \langle n \rangle}{\partial t} + \frac{\partial}{\partial x} \langle nu \rangle = 0$$

we find that the potential ϕ must satisfy the following equation

$$\frac{\partial \varphi}{\partial t} + V_0(\varphi)\frac{\partial \varphi}{\partial x} = 0, \qquad (6)$$

where $V_0 = \frac{\Phi'_\phi}{F'_\phi}$ is a certain velocity of longitudinal waves in the plasma which depends on the potential ϕ. It can be readily seen that the specific solutions depending on $x - V_0 t$, where V_0 = const, are special cases of equation (6) in the case of $V_0(\phi)$ = const. /150

From this point on, it will be assumed that the function $V_0(\phi)$ is given, and it may be employed to formulate the solutions both for the equations of the hydrodynamic approximation and for the kinetic equation.

The equations of the hydrodynamic approximation

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{\partial \varphi}{\partial x} - \frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (7)$$

may be reduced to equation (6) and to the equation for hydrodynamic velocity $v(x, t)$. In actuality, since

$$v = \frac{\langle nu \rangle}{\langle n \rangle},$$

it follows from expressions (3) and (4) that $v = v(\phi)$. Therefore, the Navier-Stokes equation (Ref. 7) has the following form

$$\frac{dv}{dt}\left[\frac{\partial \varphi}{\partial t} + v\frac{\partial \varphi}{\partial x}\right] = -\frac{\partial \varphi}{\partial x} - \frac{1}{\rho}\frac{dp}{d\varphi}\cdot\frac{\partial \varphi}{\partial x}$$

and is identical to equation (6), if only $v(\phi)$ is determined by the equation

$$\frac{v \cdot v'_\varphi + 1 + \frac{1}{\rho}p'_\varphi}{v'_\varphi} = V_0(\varphi). \qquad (8)$$

The electron plasma density $\langle n \rangle$ may be found from the specific wave velocity $V_0(\phi)$ and the hydrodynamic velocity $v(\phi)$ from equation (8), by means of the following relationship

$$F(\varphi) = n_0 e^{-\int\limits_{\varphi_0}^{\varphi}\frac{v'_\varphi}{v(\varphi) - V_0(\varphi)}d\varphi},$$

which follows from the equation of continuity. Thus, the dependence of the main hydrodynamic plasma parameters on the potential ϕ is always formulated in quadratures for a certain $V_0(\phi)$.

The potential of the self-consistent field ϕ is determined according to equations (6) and (2). The general solution is found in the following form from equation (6) according to a certain function $V_0(\phi)$

$$\varphi = \varphi(c),$$

where $c = c(x, t)$ is the equation of its characteristic. The characteristics of a quasilinear equation in partial derivatives (6) may be compiled according to the well known method (Ref. 5).

We shall regard t, x and ϕ as functions of a certain parameter s. /151
Then the parametric equation of characteristics can be written as follows

$$\frac{dt}{ds} = 1; \quad \frac{dx}{ds} = V_0(\varphi); \quad \frac{d\varphi}{ds} = 0,$$

from which it follows that ϕ equals ϕ_0 and does not depend on s. Therefore, $x = V_0(\phi)s + x_0$ and $t = s + t_0$, where x_0, t_0 and ϕ_0 are the values of t, x and ϕ on a certain line $s = 0$ through which the characteristics pass. In particular, if primary interest is directed toward the development of moving waves, which is caused by the nonlinearity of the medium, x_0, t_0 and ϕ_0 must characterize a given initial moving wave.

Let us assume that at the initial stage of the process there is a given moving wave with a constant phase velocity w_0. We then have

$$t = \tau; \quad x_0 = w_0\tau, \quad \varphi_0 = \varphi_0(\tau),$$

where $\phi_0(\tau)$ characterizes the initial dependence of the field potential on time. In this case, we have

$$t = s + \tau; \quad x = V_0(\varphi_0(\tau))\, s + w_0\tau; \quad \varphi = \varphi_0(\tau)$$

and after excluding s we find τ from the following equation

$$x = V_0(\varphi(\tau))(t - \tau) + w_0\tau,$$

which determines the characteristics of the quasilinear equation (6) $\tau = \tau(x, t)$. Using the characteristics from equation (2), let us determine the field potential in the plasma $\phi(x, t)$ and all of the hydrodynamic parameters of the medium.

We can find the electron velocity distribution function from the kinetic equation (1). Actually, since equation (1) is transformed into the following form, together with equation (6),

$$[u-V_0(\varphi)]\frac{\partial f}{\partial \varphi}-\frac{\partial f}{\partial u}=0,$$

the equation of characteristics

$$\frac{d\varphi}{du}=-u+V_0(\varphi)$$

enables us to find the particle velocity distribution function in a general form from the potential ϕ and the kinetic velocity u. Since the potential ϕ may be compiled within the framework of the simpler hydrodynamic approximation, the specific dependence of the distribution function on the coordinates and time has been established.

Propagation of Waves in Media with Linear Dispersion /152

We shall assume that the given law for the dependence $V_0(\phi)$ of the nonlinear wave phase velocity on potential determines the law of the medium dispersion. We shall study nonlinear, nonstationary waves in a medium with linear dispersion

$$V_0(\varphi)=V_0+\gamma\varphi,$$

where V_0 and γ are certain constant parameters.

We shall disregard the plasma pressure, so that equation (8) assumes the following form

$$\frac{d\varphi}{dv}=-v+V_0+\gamma\varphi.$$

The dependence of the hydrodynamic velocity v of the plasma on the potential can be determined according to the following relationship

$$\varphi=ce^{\gamma\chi}+\frac{\gamma\chi+1}{\gamma^2},$$

where $\chi = v - V_0$; c -- integration constant. In the case of $v = V_0$, dispersion disappears, i.e., in the sense that V_0 is the limiting plasma velocity at which the electric field potential vanishes. We then have

$$\varphi=\frac{1}{\gamma^2}[(\gamma\chi+1)-e^{\gamma\chi}]. \quad (9)$$

Since

$$\frac{dv}{d\varphi} = -\frac{1}{v - V_0(\varphi)},$$

the electron plasma density may be found according to formula (9)

$$n = n_0 e^{\varphi_0 \int^{\varphi} \frac{d\varphi}{[v - V_0(\varphi)]^2}} = n_0 e^{\int_{\chi_0}^{\chi} \frac{\frac{d\varphi}{d\chi}}{[\chi - \gamma\varphi]^2} d\chi}.$$

Since

$$\int_{\chi_0}^{\chi} \frac{\frac{d\varphi}{d\chi}}{[\chi - \gamma\varphi]^2} d\chi = \int_{\chi_0}^{\chi} \frac{d(\gamma\chi)}{1 - e^{\gamma\chi}} = \ln \frac{e^{\gamma\chi}}{1 - n^{\gamma\chi}},$$

$$F(\varphi) = n_0 \frac{e^{\gamma\chi}}{1 - e^{\gamma\chi}},$$

where χ_0 is the value $v = V_0$ at which the electron density equals the ion density.

Consequently, the Poisson equation can be written as follows /153

$$\frac{\partial^2\varphi}{\partial x^2} = n_0^2 \frac{1}{1 - e^{\gamma\chi}},$$

and χ may be found from relationship (9).

Let us determine the characteristic of equation (6) for the linear dispersion law

$$x = [V_0 + \gamma\varphi_0(\tau)](t - \tau) + w_0\tau.$$

Let us assume that at an initial moment of time the field is switched on whose potential increases linearly with time:

$$\varphi_0(\tau) = \varphi_0 \frac{\tau}{T}.$$

We then have

$$\tau^2 + \left\{\frac{T}{\varphi_0\gamma}(V_0 - w_0) - t\right\}\tau + \frac{T}{\varphi_0\gamma}(x - V_0 t) = 0,$$

from which it follows that

$$\tau = \frac{1}{2}\left[t + \frac{T}{\varphi_0\gamma}(w_0 - V_0)\right] \pm$$
$$\pm \frac{1}{2}\sqrt{\left[t + \frac{T}{\varphi_0\gamma}(w_0 - V_0)\right]^2 - \frac{4T}{\varphi_0\gamma}(x - V_0 t)}.$$

Consequently, the characteristic is real only up to a certain value x_{bound} which depends on time

$$x_{bound} = V_0 t + \frac{\varphi_0\gamma}{4T}\left[t + \frac{T}{\varphi_0\gamma}(w_0 - V_0)\right]^2.$$

If $x > x_{bound}$, the signal which is switched on in the case of $\tau = 0$ has still not reached the point under consideration. If $x < x_{bound}$, the characteristic is determined in more than one way (it has two values). The requisite value of the characteristic is determined by additional considerations.

The propagation rate of the front boundary is

$$V_{bound} = \frac{dx_{bound}}{dt} = \frac{V_0 + w_0}{2} + 2\frac{\varphi_0\gamma}{4T}t.$$

Consequently, for $\gamma > 0$ the front velocity increases with the time, while in the case of $\gamma < 0$ it decreases.

The velocity of a point with a constant given potential, as may be seen from (9), is determined by the wave phase velocity $V_0(\phi)$, i.e., by the specific value of the potential ϕ. We may formulate the potential and the electron velocity distribution function by the hydrodynamic parameters which are found.

Electron Velocity Distribution Function in the Case of a Square Dispersion Law /154

Let us find the electron velocity distribution function in the case of media with a square dispersion law

$$V_0(\varphi) = V_0 \pm \sqrt{-2\varphi}.$$

The equation of characteristics

$$\frac{d\varphi}{d\chi} = -\chi_1 \pm \sqrt{-2\varphi},$$

where $\chi_1 = u - V_0$; u -- electron kinetic velocity in the plasma, can be integrated by substituting $\phi = \chi_1^2 t$.

We finally find

$$\frac{m(u-V_0)^2}{2kT}\left[\frac{-2\varphi}{(u-V_0)^2}+\sqrt{\frac{-2\varphi}{(u-V_0)^2}-1}\right]\times$$
$$\times\left[\frac{1+\dfrac{2\sqrt{\dfrac{-2\varphi}{(u-V_0)^2}}}{1+\sqrt{5}}}{1-\dfrac{2\sqrt{\dfrac{-2\varphi}{(u-V_0)^2}}}{\sqrt{5}-1}}\right]^{\frac{1}{\sqrt{5}}}=C_1=\text{const} \tag{10}$$

for a positive sign of the root, and

$$\frac{m(u-V_0)^2}{2kT}\left[\frac{-2\varphi}{(u-V_0)^2}-\sqrt{\frac{-2\varphi}{(u-V_0)^2}-1}\right]\times$$
$$\times\left[\frac{1-\dfrac{2\sqrt{\dfrac{-2\varphi}{(u-V_0)^2}}}{\sqrt{5}+1}}{1+\dfrac{2\sqrt{\dfrac{-2\varphi}{(u-V_0)^2}}}{\sqrt{5}-1}}\right]^{\frac{1}{\sqrt{5}}}=C_2=\text{const} \tag{11}$$

for a negative sign of the root.

The solutions of (10) and (11) enable us to compile the distribution /155 functions in the case of $\phi \to 0$ which change into a Maxwell distribution. On the other hand, all the distributions changing into Maxwell distributions in the case of $\phi \to 0$, in regions with a non-zero electric field, have the following form

$$f = A\exp C_1 \text{ for } u > V_0;$$
$$f = A\exp C_2 \text{ for } u < V_0.$$

Consequently, electrons have velocities lying outside of the boundaries

$$u > V_0 + \frac{2\sqrt{-2\varphi}}{\sqrt{5}-1}$$

and

$$u < V_0 - \frac{2\sqrt{-2\varphi}}{\sqrt{5}+1}.$$

The electrons whose velocities are included within

$$V_0 - \frac{2\sqrt{-2\varphi}}{\sqrt{5}+1} < u < V_0 + \frac{2\sqrt{-2\varphi}}{\sqrt{5}-1},$$

are damped by a wave, and are not included in the distribution functions of (10) and (11). The relationships obtained enable us to study the collisionless transition of electrons from the region of trapped particles into other plasma electrons and the associated energy distribution.

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INDUCED SCATTERING OF LANGMUIR OSCILLATIONS IN A PLASMA LOCATED IN A STRONG MAGNETIC FIELD

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This article investigates the nonlinear interaction of harmonics in the long wave spectral region of Langmuir oscillations ($k\gamma_{De} \ll 1$). The linear damping of these oscillations, which is caused by the interaction with resonance particles, is negligibly small. It is assumed that the plasma is located in a rather strong magnetic field, so that the plasma particle oscillations are possible only in the direction of the magnetic field which is parallel to oz. /156

The initial system of equations for the distribution functions of electrons and ions and the electric field has the following form

$$\frac{\partial f^{\alpha}_{\vec{k}}}{\partial t} + i(k_z v_z - \omega_{\vec{k}}) f^{\alpha}_{\vec{k}} + \frac{e_\alpha}{m_\alpha} E_{\vec{k}z} \frac{\partial f^{\alpha}_0}{\partial v_z} = -\frac{e_\alpha}{m_\alpha} \sum_{\vec{q}} E_{\vec{k}-\vec{q}z} \frac{\partial f^{\alpha}_{\vec{q}}}{\partial v_z} \times$$
$$\times \exp\left\{-i\left(\omega_{\vec{k}-\vec{q}} + \omega_{\vec{q}} - \omega_{\vec{k}}\right) t\right\}; \quad (1)$$

$$i\vec{k}\vec{E}_{\vec{k}} = 4\pi \sum_{\alpha} e_\alpha \int f^{\alpha}_{\vec{k}} d\vec{v}; \quad (2)$$

$$\vec{E} = \frac{1}{2} \sum_{\vec{k}} \vec{E}_{\vec{k}} e^{i(\vec{k}\vec{r} - \omega_{\vec{k}} t)} + \text{c.c.}^{*}; \quad f^{\alpha} = \frac{1}{2} \sum_{\vec{k}} f^{\alpha}_{\vec{k}} e^{i(\vec{k}\vec{r} - \omega_{\vec{k}} t)} + \text{c.c.} \quad (3)$$

(f_0 is the background distribution function whose change with time can be disregarded, due to the small number of resonance particles $\left(\frac{v_{Te}}{v_{\Phi}} \sim k\lambda_{De} \ll 1\right)$ and the small dissipation of oscillation energy during scattering). The notation in equations (1) - (3) is standard; summation in equation (2) is performed for plasma ions and electrons. We obtained the following relationship for damping frequency and decrement, disregarding the nonlinear terms in the first equation, from equations (1) and (2):

$$\omega_{\vec{k}} = \omega_{0e}\left(1 + \frac{3}{2} k^2 \lambda^2_{De}\right) \cos\theta; \quad \gamma_{\vec{k}} = \frac{\pi}{2} \cdot \frac{\omega^3_{0e}}{k^2} \cos\theta \left. \frac{\partial f_0}{\partial v_z} \right|_{v_z = \frac{\omega_{\vec{k}}}{k_z}} \quad (4)$$

(θ is the angle between the direction of oscillation propagation and the magnetic field). The nonlinear interaction of harmonics leads to a change in the oscillation spectrum due to processes of wave decay and scattering by plasma particles. The laws of conservation must be fulfilled in the /157

* c. c. = complex conjugate.

case of two-plasma decays

$$\omega_{\vec{k}_1} + \omega_{\vec{k}_2} = \omega_{\vec{k}}; \quad \vec{k}_1 + \vec{k}_2 = \vec{k}. \qquad (5)$$

Assuming that $\vec{k}_1$ and $\vec{k}_2$ lie in one plane, we obtain the following condition for the spectrum (4) from (5):

$$\frac{k_1 \cos\theta_1 \pm k_2 \cos\theta_2}{\sqrt{k_1^2 + k_2^2 \pm 2k_1 k_2 \cos(\theta_1 - \theta_2)}} = \cos\theta_1 \pm \cos\theta_2. \qquad (6)$$

Decay is possible if $k_{1z} > 0$, $k_{2z} < 0$, which corresponds to the sign "--" in condition (6). In several cases, the spectrum of Langmuir oscillations in a strong magnetic field is a nondecay spectrum, particularly if it is close to a one-dimensional spectrum $\theta_1 \simeq \theta_2$*.

Nonlinear wave scattering is caused by the interaction of plasma particles with the beats of different frequency. This process becomes significant if the condition $\frac{\omega_{\vec{k}_1} - \omega_{\vec{k}_2}}{k_{1z} - k_{2z}} \lesssim v_{Te}$. is fulfilled. The beats which are the cause of wave scattering cannot arise due to decay, since the following condition is fulfilled for the waves formed during the decay

$$\frac{\omega_{\vec{k}_1} - \omega_{\vec{k}_2}}{k_{1z} - k_{2z}} = \frac{\omega_{\vec{k}}}{k_z} = \frac{\omega_0}{|\vec{k}_1 - \vec{k}_2|} \gg v_{Te}.$$

Therefore, in this case the processes of wave decay and scattering are independent. The transformation of the oscillation spectrum due to the nonlinear scattering process will be subsequently investigated.

In solving the nonlinear equation (1), we shall employ the method of perturbations. Substituting $f^{\alpha}_{\vec{k}}$, which is found from the linear theory, in the nonlinear terms of this equation, we may employ (2) to obtain the following formulas for the electric field amplitude and the distribution function in the second approximation /158

* Ion oscillation branches are not examined. Actually, in the approximation under consideration, decay of a Langmuir wave into a Langmuir ion-sound wave is possible. However, the energy primarily remains in high-frequency oscillations (Ref. 1).

$$\vec{E}^{(2)}_{\vec{k}} = i\sum_{\alpha}\frac{4\pi e_\alpha^3}{m_\alpha^2}\cdot\frac{\vec{k}}{k^2}\sum_{\vec{q}}\frac{E_{\vec{k}-\vec{q}z}E_{\vec{q}z}\exp\left\{-i\left(\omega_{\vec{k}-\vec{q}}+\omega_{\vec{q}}-\omega_{\vec{k}}\right)t\right\}}{\varepsilon\left(\omega_{\vec{k}-\vec{q}}+\omega_{\vec{q}},\vec{k}\right)}\times$$

$$\times\int\frac{1}{k_zv_z-\omega_{\vec{k}-\vec{q}}-\omega_{\vec{q}}}\cdot\frac{\partial}{\partial v_z}\left(\frac{\frac{\partial f_0^\alpha}{\partial v_z}}{q_zv_z-\omega_{\vec{q}}}\right)d\vec{v};$$

$$f^{(2)\alpha}_{\vec{k}} = -\sum_{\vec{q}}\frac{E_{\vec{k}-\vec{q}z}E_{\vec{q}z}\exp\left\{-i\left(\omega_{\vec{k}-\vec{q}}+\omega_{\vec{q}}-\omega_{\vec{k}}\right)t\right\}}{k_zv_z-\omega_{\vec{k}-\vec{q}}-\omega_{\vec{q}}}\times \tag{7}$$

$$\times\left[\frac{e_\alpha^2}{m_\alpha^2}\cdot\frac{\partial}{\partial v_z}\left(\frac{\frac{\partial f_0^\alpha}{\partial v_z}}{q_zv_z-\omega_{\vec{q}}}\right)+\frac{e_\alpha}{m_\alpha}\cdot\frac{k_z}{k^2}\cdot\frac{\partial f_0^\alpha}{\partial v_z}\cdot\frac{1}{\varepsilon\left(\omega_{\vec{k}-\vec{q}}+\omega_{\vec{q}},\vec{k}\right)}\times\right.$$

$$\left.\times\sum_{\beta}\frac{4\pi e_\beta^3}{m_\beta^2}\int\frac{1}{k_zv_z-\omega_{\vec{k}-\vec{q}}-\omega_{\vec{q}}}\cdot\frac{\partial}{\partial v_z}\left(\frac{\frac{\partial f_0^\beta}{\partial v_z}}{q_zv_z-\omega_{\vec{q}}}\right)\right]d\vec{v}.$$

In the third approximation, the system of equations for determining the electric field amplitude and the distribution function has the following form

$$\frac{\partial f^\alpha_{\vec{k}}}{\partial t}+i\left(k_zv_z-\omega_{\vec{k}}\right)f^\alpha_{\vec{k}}+\frac{e_\alpha}{m_\alpha}E_{\vec{k}z}\frac{\partial f_0^\alpha}{\partial v_z}=-\frac{e_\alpha}{m_\alpha}\sum_{\vec{q}}\left(E^{(1)}_{\vec{k}-\vec{q}z}\frac{\partial f^{(2)}_{\vec{q}}}{\partial v_z}+\right.$$

$$\left.+E^{(2)}_{\vec{k}-\vec{q}z}\frac{\partial f^{(1)}_{\vec{q}}}{\partial v_z}\right)\exp\left\{-i\left(\omega_{\vec{k}-\vec{q}}+\omega_{\vec{q}}-\omega_{\vec{k}}\right)t\right\}; \tag{8}$$

$$i\vec{k}\vec{E}_{\vec{k}}=4\pi\sum_\alpha e_\alpha\int f^\alpha_{\vec{k}}d\vec{v},$$

where $E^{(2)}_{\vec{k}}$, $f^{(2)}_{\vec{k}}$ are determined from formulas (7). We obtain the kinetic equation for waves from these equations by simple computations: /159

$$\frac{\partial E_{\vec{k}z}}{\partial t}=\gamma_{\vec{k}}E'_{\vec{k}z}-i\sum_{\vec{q},\vec{\varkappa}}H\left(\vec{k},\vec{q},\vec{\varkappa}\right)E_{\vec{k}-\vec{q}z}E_{\vec{q}-\vec{\varkappa}z}\times \tag{9}$$

$$\times\exp\left\{-i\left(\omega_{\vec{k}-\vec{q}}+\omega_{\vec{q}-\vec{\varkappa}}+\omega_{\vec{\varkappa}}-\omega_{\vec{k}}\right)t\right\},$$

where $\gamma_{\vec{k}}$ is the decrement of Landau linear damping:

$$H(\vec{k},\vec{q},\vec{\varkappa}) = -\frac{k_z}{k^2}\cdot\frac{\omega_{\vec{k}-\vec{q}}+\omega_{\vec{q}-\vec{\varkappa}}+\omega_{\vec{\varkappa}}-\omega_{\vec{k}}}{\varepsilon\left(\omega_{\vec{k}-\vec{q}}+\omega_{\vec{q}-\vec{\varkappa}}+\omega_{\vec{\varkappa}};\vec{k}\right)}\times$$

$$\times\left\langle\sum_\alpha\frac{4\pi e_\alpha^4}{m_\alpha^3}\int\frac{1}{k_z v_z-\omega_{\vec{k}-\vec{q}}-\omega_{\vec{q}-\vec{\varkappa}}-\omega_{\vec{\varkappa}}}\times\right.$$

$$\times\frac{\partial}{\partial v_z}\left[\frac{1}{q_z v_z-\omega_{\vec{q}-\vec{\varkappa}}-\omega_{\vec{\varkappa}}}\cdot\frac{\partial}{\partial v_z}\left(\frac{\frac{\partial f_0^\alpha}{\partial v_z}}{\varkappa_z v_z-\omega_{\vec{\varkappa}}}\right)\right]d\vec{v}+$$

$$+\frac{q_z}{q^2}\cdot\frac{1}{\varepsilon\left(\omega_{\vec{q}-\vec{\varkappa}}+\omega_{\vec{\varkappa}};\vec{q}\right)}\sum_\alpha\frac{4\pi e_\alpha^3}{m_\alpha^2}\int\frac{1}{k_z v_z-\omega_{\vec{k}-\vec{q}}-\omega_{\vec{q}-\vec{\varkappa}}-\omega_{\vec{\varkappa}}}\times$$

$$\times\frac{\partial}{\partial v_z}\left[\left(\frac{1}{q_z v_z-\omega_{\vec{q}-\vec{\varkappa}}-\omega_{\vec{\varkappa}}}+\frac{1}{(k_z-q_z)v_z-\omega_{\vec{k}-\vec{q}}}\right)\frac{\partial f_0^\alpha}{\partial v_z}\right]\times$$

$$\left.\times\sum_\alpha\frac{4\pi e_\beta^3}{m_\beta^2}\int\frac{1}{q_z v_z-\omega_{\vec{q}-\vec{\varkappa}}-\omega_{\vec{\varkappa}}}\cdot\frac{\partial}{\partial v_z}\left(\frac{\frac{\partial f_0^\alpha}{\partial v_z}}{\varkappa_z v_z-\omega_{\vec{\varkappa}}}\right)d\vec{v}\right\rangle.$$

Multiplying (9) by $E^*_{\vec{k}z}$, averaging over time (only the terms with /160
$\vec{\varkappa}=\vec{k}$ and $\vec{\varkappa}=\vec{q}-\vec{k}$, which change slowly with time, remain in the right-hand side) and combining the equation obtained with the complex conjugate one, we can write

$$\frac{\partial}{\partial t}\left|E_{\vec{k}z}\right|^2 = 2\gamma^l_{\vec{k}}\left|E_{\vec{k}z}\right|^2+2\Sigma_{\vec{q}}\gamma^{ll}_{\vec{k}\vec{q}}\left|E_{\vec{k}z}\right|^2\left|E_{\vec{q}}\right|^2 \qquad (11)$$

$$\gamma^{ll}_{\vec{k}} = \frac{q_z^2}{q^2}\,\mathrm{Im}\,\left\langle H(\vec{k},\vec{k}-\vec{q},\vec{q})+H(\vec{k},\vec{k}-\vec{q},\vec{k})\right\rangle =$$

$$=\frac{k_z^2}{k^2}\cdot\frac{1}{\frac{\partial\varepsilon\left(\omega_{\vec{k}},\vec{k}\right)}{\partial\omega_{\vec{k}}}}\,\mathrm{Im}\,\left\langle\sum_\alpha\frac{4\pi e_\alpha^4}{m_\alpha^3}(k_z-q_z)\int\frac{\frac{\partial f_0^\alpha}{\partial v_z}}{\left(k_z v_z-\omega_{\vec{k}}\right)^3}\times\right.$$

$$\times\frac{1}{\left(q_z v_z-\omega_{\vec{q}}\right)\left[(k_z-q_z)v_z-\omega_{\vec{k}}+\omega_{\vec{q}}\right]}d\vec{v}+\frac{(k_z-q_z)^2}{\left|\vec{k}-\vec{q}\right|^2}\times$$

$$\times \frac{1}{\varepsilon\left(\omega_{\vec{k}} - \omega_{\vec{q}}, \vec{k} - \vec{q}\right)} \times$$

$$\times\left(\sum_{\alpha} \frac{4\pi e_\alpha^3}{m_\alpha^2} \int \frac{\frac{\partial f_0^\alpha}{\partial v_z}}{\left(k_z v_z - \omega_{\vec{k}}\right)\left(q_z v_z - \omega_{\vec{q}}\right)\left[\left(k_z - q_z\right) v_z - \omega_{\vec{k}} + \omega_{\vec{q}}\right]} d\vec{v}\right)^2 \rangle . \qquad (12)$$

In computing the integrals included in equation (12), we assume that the following conditions are fulfilled: $\omega_{\vec{k}} >> kv_T$, which corresponds to weak wave absorption, and $\omega_{\vec{k}} \simeq \omega_{\vec{q}}$, which is necessary in order that a significant number of plasma particles may participate in the interaction with beats of different frequency. Let us investigate the case when the contribution made by ions to the plasma polarizability at a different frequency is negligibly small -- i.e., the condition $\frac{\omega_{0i}^2}{(\omega_{\vec{k}} - \omega_{\vec{q}})^2} \sim$ $\sim \frac{m_e}{m_i k^4 \lambda_{De}^4} \ll 1$. is fulfilled. Then, assuming that the distribution func- /161 tion of the plasma particles f_0^α is a Maxwell distribution, we obtain the following from equation (12) after very cumbersome computations

$$\gamma^{ll}(k, \theta, q, \theta') = -\frac{27}{4}\sqrt{\frac{\pi}{2}} \cdot \frac{e^2}{m_e^2 \omega_{0e}^3} \cos\theta k^2 q^2 \frac{k^2 - q^2}{|k - q|} \lambda_{De}^3 \times$$
$$\times \left\{1 + \frac{2}{3} \cdot \frac{(\cos\theta - \cos\theta')}{(k^2 - q^2)\lambda_{De}^2 \cos\theta}\right\} \exp\left\{-\frac{m}{2T_e} \cdot \frac{\omega_{0e}^2 (\cos\theta - \cos\theta')^2}{(k - q)^2 \cos^2\theta}\right\}. \qquad (13)$$

The increment $\gamma^{\ell\ell}(k,\theta,q,\theta')$ which determines the induced scattering rate of plasma oscillations in k-space differs considerably from zero only in the narrow range of angles: $|\theta' - \theta| \lesssim \frac{v_{Te}}{v_\Phi}$ in the case of $\theta \neq 0$ and $|\theta - \theta'| \leqslant \sqrt{\frac{v_{Te}}{v_\Phi}}$ in the case of $\theta \simeq 0$.

Thus, during scattering the spectrum which is initially one-dimensional remains close to a one-dimensional spectrum. In the case of $\theta' - \theta\ \gamma^{\ell\ell}$, it coincides with the increment obtained previously in a one-dimensional model, within an accuracy of the factor cos θ (Ref. 2, 3). It increases somewhat with an increase in $\theta - \theta'\ \gamma^{\ell\ell}$ -- up to $\gamma^{ll}_{max} \approx \frac{2}{3\sqrt{e}} \cdot \frac{1}{(k+q)\lambda_{De}} \gamma^{ll}(\theta = \theta')$, and then rapidly decreases to 0.

In the real case of a confined plasma the spectrum of the values θ is always discreet, and for a sufficiently small ratio $\frac{v_{Te}}{v_\Phi}$, only one possible value $\theta' = \theta$ can enter the angular range $\theta' - \theta$ in which $\gamma^{\ell\ell}$ is large. The scattering of plasma oscillations in a strong magnetic field then takes place, in fact, in the same way as in a one-dimensional model, in contrast to scattering of oscillations in a plasma which is not located in a magnetic field. In the latter case, scattering at large angles is possible, and, if the angle between the wave vectors of two interacting waves considerably exceeds $\frac{v_{Te}}{v_\Phi}$, $\gamma^{\ell\ell}$ increases by a factor of $\frac{1}{k^2\lambda_{De}^2}$ as compared with $\gamma^{\ell\ell}$ in a one-dimensional model (Ref. 4, 5). The contribution made by ions to the plasma dielectric constant is quite significant when the condition $\frac{\omega_{0i}^2\cos^2\theta}{\left(\omega_{\vec{k}} - \omega_{\vec{q}}\right)^2} \gg 1$ is fulfilled. We should note that this condition is fulfilled most readily in the case of $\theta \simeq \theta$, when the difference $\omega_{\vec{k}} - \omega_{\vec{q}}$ is at a minimum: $\omega_{\vec{k}} - \omega_{\vec{q}} \simeq \frac{3}{2}\omega_{0e}\cos\theta\,(k^2 - q^2)\lambda_{De}^2$ and /162 $\frac{\omega_{0i}^2\cos^2\theta}{\left(\omega_{\vec{k}} - \omega_{\vec{q}}\right)^2} \sim \frac{m_e}{m_i}\cdot\frac{1}{k^4\lambda_{De}^4}$. Assuming that the conditions $\frac{m_e}{m_i}\cdot\frac{1}{k^4\lambda_{De}^4} \gg 1$ и $\frac{m_e}{m_i} \times \times \frac{1}{k^2\lambda_{De}^2} \ll 1$, are fulfilled, we obtain the following expression from (12) in the case of $\theta = \theta'$ for $\gamma^{\ell\ell}$

$$\gamma^{ll}(k, \theta, q, \theta') = -\frac{4}{27}\sqrt{\frac{\pi}{2}}\cdot\frac{e^2}{m_e^2\omega_{0e}^3}\left(\frac{m_e}{m_i}\right)^2\frac{k^2 - q^2}{|k - q|}\cdot\frac{\cos\theta}{(k+q)^4\lambda_{De}^5}, \qquad (14)$$

i.e., in this case $\gamma^{\ell\ell}$ increases somewhat as compared with (13). In the case of $\theta \neq \theta'$ $\left(\cos\theta - \cos\theta' \sim \frac{v_{Te}}{v_\Phi}\right)\frac{\omega_{0i}^2}{\left(\omega_{\vec{k}} - \omega_{\vec{q}}\right)^2} \sim \frac{m_e}{m_i} \times \frac{1}{k^2\lambda_{De}^2}$ and when the condition $\frac{m_e}{m_i}\cdot\frac{1}{k^2\lambda_{De}^2} \ll 1$ is fulfilled, $\gamma^{\ell\ell}$ is determined in this region by formula (13).

We should recall that this investigation pertains to a case when the plasma is located in a magnetic field which is so strong that the plasma particle oscillations are only possible in the direction of the magnetic field. The condition of "magnetization" of the electron component, as is customary, has the form $\omega_{0e} \ll \omega_{He}$, i.e., it is fulfilled for field strengths which are not too large. In this case, when the ions make a significant contribution to the dielectric constant of the plasma, it must be required that the ion plasma component be "magnetized". The corresponding condition is harder:

$$\left|\omega_{\vec{k}} - \omega_{\vec{q}}\right| \ll \omega_{Hi} \text{ i.e. } \omega_{He} \gg \omega_{0e} \frac{m_i}{m_e} k^2 \lambda_{De}^2.$$

Let us turn to certain general characteristics of the nonlinear change in the wave spectrum. Since the increment of nonlinear scattering $\gamma^{\ell\ell}(k,\theta,q,\theta')$ in the one-dimensional case $\theta = \theta'$ changes sign when $\vec{k} \rightleftarrows \vec{q}$ is substituted [see formulas (13), (14)], the change in the total oscillation energy during scattering in the approximation under consideration equals zero

$$\frac{d}{dt} \sum_{\vec{k}} \left|E_{\vec{k}}\right|^2 = \sum_{\vec{k}, \vec{q}} \gamma^{ll}(k, 0, q, \theta') \left|E_{\vec{k}}\right|^2 \left|E_{\vec{q}}\right|^2 = 0. \tag{15}$$

Employing formulas (13), (14), we may also readily see that the nonlinear interaction of harmonics leads to a transfer of energy along the spectrum to smaller k: /163

$$\frac{d}{dt} \sum_{\vec{k}} k \left| E_{\vec{k}}\right|^2 = \sum_{\vec{k}, \vec{q}} \frac{1}{2}(k-q)\gamma^{ll}(k, \theta, q, \theta') \left|E_{\vec{k}}\right|^2 \left|E_{\vec{q}}\right|^2 < 0. \tag{16}$$

The total oscillation energy during scattering by plasma particles changes in the subsequent series with respect to $k^2\lambda_{De}^2$, since in this approximation an addition to $\gamma^{\ell\ell}$ appears, which is symmetrical with respect to the $\vec{k} \rightleftarrows \vec{q}$ substitution. Assuming, for purposes of simplicity, that the oscillation spectrum is one-dimensional, we obtain the equation for the change in the total energy in the spectrum:

$$\frac{\partial}{\partial t} \sum_{\vec{k}} \left|E_{\vec{k}}\right|^2 \simeq -\sum_{\vec{k}} \sqrt{\frac{\pi}{2}} \cdot \frac{\omega_{\vec{k}}}{k^3 \lambda_{De}^3} e^{-\frac{1}{k^2\lambda^2_{De}} - \frac{3}{2}} \left|E_{\vec{k}}\right|^2 - \tag{17}$$

$$- \frac{243}{4} \sqrt{\frac{\pi}{2}} \cdot \frac{e^2}{m^2} \cdot \frac{\lambda_{De}^5}{\omega_{0e}^3} \sum_{\vec{k}, \vec{q}} k^2 q^2 |k-q| \left|E_{\vec{k}}\right|^2 \left|E_{\vec{q}}\right|^2.$$

The second term in this equation which describes the oscillation energy change during scattering becomes more significant than the first term, which characterizes the oscillation energy change as a result of their interaction with resonance particles, when the following condition is fulfilled

$$\frac{\sum_q |E_q|^2}{4\pi N_0 T} \gtrsim \frac{1}{60} e^{-\frac{3}{2}} \frac{e^{-\frac{1}{2k^2\lambda_{De}^2}}}{k^{10}\lambda_{De}^{10}}, \tag{18}$$

i.e., for relatively small oscillation amplitudes $|E_k|^2 \ll N_0T$, if the parameter $\frac{v_{Te}}{v_\Phi}$ is fairly small.

The possible dissipation of oscillation energy when they undergo nonlinear scattering by plasma particles was pointed out in (Ref. 6, 7). However, for Langmuir oscillations this phenomenon is $k^2\lambda_{De}^2$ times less than the change in the field intensity in separate harmonics in the spectrum due to energy transfer.

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NONLINEAR THEORY OF LOW FREQUENCY OSCILLATIONS EXCITED BY AN ION BUNDLE IN A PLASMA

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As is well known, in the case of the interaction between a bundle of rapid electrons and a plasma, the energy lost by the bundle during relaxation changes into thermal energy of electrons in the plasma and in the bundle, and into energy of high frequency Langmuir oscillations (Ref. 1). When investigating the possibility of heating the plasma ion component during bunched instabilities, it is of interest to investigate the excitation of low frequency oscillations by the bundle, in the nonlinear approximation. A previous article by D. G. Lominadze (in collaboration with K. N. Stepanov) investigated the linear theory of low frequency oscillation excitation in a plasma located in a magnetic field by an ion bundle. In a strongly non-isothermic plasma ($T_e >> T_i$), during the passage of an ion bundle, longitudinal longwave $\left(\frac{k_\perp v_{\perp\alpha}}{\omega_{H\alpha}} << 1\right)$ oscillations may be excited, whose frequency is determined by the following relationship

$$\omega_\pm^2 = \frac{1}{2}\left(\omega_s^2 + \omega_{Hi}^2\right) \pm \frac{1}{2}\left[\left(\omega_s^2 + \omega_{Hi}^2\right)^2 - 4\omega_s^2\omega_{Hi}^2\cos^2\theta\right]^{1/2}, \tag{1}$$

where

$$\omega_s^2 = \frac{\omega_{0i}^2}{1 + \frac{1}{k^2\lambda_{De}^2}};\quad \omega_{0i}^2 = \frac{4\pi n_0 e^2}{M};\quad \lambda_{De}^2 = \frac{T_e}{4\pi n_0 e^2};$$

$\omega_{Hi,e}$ is the Larmor frequency of ions and electrons, respectively; θ -- /165
the angle between $\vec{H}_0$ and the direction of oscillation propagation. If $T_i \sim T_e$, the ion bundle can excite longitudinal shortwave oscillations $\left(\frac{k_\perp v_{Ti}}{\omega_{Hi}} \sim 1\right)$ in the plasma with the frequencies (Ref. 2)

$$\omega_n = n\omega_{Hi}(1 + \psi_n);\quad \psi_n = \frac{I_n(\mu_i)}{\left(1 + \frac{T_i}{T_e}\right)e^{\mu_i} - \sum\limits_{m=-\infty}^{\infty}{}' \frac{nI_n(\mu_i)}{n-m}}, \tag{2}$$

$$n = 1, 2, \ldots,$$

where $\mu_i = \frac{k_\perp^2 v_{Ti}}{\omega_{Hi}^2}$; $I_n(\mu_i)$ is the Bessel function of the imaginary argument.

The process by which bunched instabilities develop may be qualitatively divided into two stages (Ref. 1). In the initial (hydrodynamic) stage, the bundle remains monoenergetic $\left(\frac{k_z v_{T1}}{\gamma_{\vec{k}}} \ll 1,\ v_{T1}\right.$ -- thermal velocity of bundle ions$\left.\right)$, and instability develops very rapidly; $\tau_r \sim \left(\frac{n_0}{n_1}\right)^{1/3} \frac{1}{\omega}$ (n_0, n_1 -- densities of plasma and bundle, $n_1 \ll n_0$, ω -- excitable frequency). For rather large oscillation amplitudes, the thermal energy in the bundle is so large $\left(\frac{k_z v_{T_1}}{\gamma_{\vec{k}}} \sim 1\right)$, that the bundle relaxation may be investigated in the quasilinear approximation (quasilinear stage). The time required for the development of this instability τ_{quasi} is on the order of $\frac{n_0}{n_1} \cdot \frac{1}{\omega}$.

This article investigates the development of instability at the first stage, the most unstable oscillation branches are found which produce the dynamics of the instability development, and the change in the macroscopic parameters of the bundle and the plasma is determined (thermal energy, directed velocity) in the case of instability. In a strong magnetic field ($\omega_{Hi} \gg \omega_s$) it is possible to trace the development of instability at the quasilinear stage and to determine the state at which the bundle and the plasma arrive as a result of the quasilinear relaxation process.

Excitation of Longwave, Low Frequency Oscillations (Hydrodynamic Stage) /166

We shall assume that at this stage the following condition is fulfilled

$$\frac{k_z v_{T1}}{\gamma_{\vec{k}}} \ll 1; \frac{k_z \delta u}{\gamma_{\vec{k}}} \ll 1 \tag{3}$$

(δu is the change in the bundle velocity at the initial stage), at which the dispersion equation of longwave oscillations has the form

$$1 + \frac{1}{k^2\lambda_{De}^2} - \frac{\omega_{0i}^2}{\omega^2}\cos^2\theta - \frac{\omega_{0i}^2}{\omega^2 - \omega_{Hi}^2}\sin^2\theta - \frac{\omega_{01}^2}{(\omega - k_z u_0)^2}\cos^2\theta - \frac{\omega_{01}^2}{(\omega - k_z u_0)^2 - \omega_{Hi}^2}\sin^2\theta = 0. \tag{4}$$

When investigating the dispersion equation (4), we can examine two cases.

1. $\omega_{Hi} \ll \omega_s$ (weak magnetic field). During Cherenkov excitation ($\omega = k_z u_0$), oscillations with the frequency

$$\omega \simeq \omega_s \left(1 + \frac{1}{2} \cdot \frac{\omega_{Hi}^2}{\omega_s^2} \sin^2 \theta\right).$$

have the largest increasing amplitude increment. The maximum value of the corresponding increment is

$$\eta = \frac{\sqrt{3}}{2^{4/3}} \left(\frac{n_1}{n_0}\right)^{1/3} \omega_s \cos^{2/3} \theta. \tag{5}$$

The formation of instabilities in this case is possible in the case of $\cos \theta < \frac{v_s}{u_0}$, if $u_0 > v_s$, and for any $\cos \theta$, if $u_0 < v_s \left(v_s = \sqrt{\frac{T_e}{M}}\right)$.

2. $\omega_{Hi} \gg \omega_s$ (strong magnetic field). In this case, ion-sound waves with the frequency $\omega = \omega_s \cos \theta$ have the largest increment. The increasing increment of these waves is

$$\varepsilon = \frac{\sqrt{3}}{2^{4/3}} \left(\frac{n_1}{n_0}\right)^{1/3} \omega_s \cos \theta. \tag{6}$$

Excitation of oscillations is possible if $\frac{v_s}{u_0} > 1$.

Relationships (4) - (6) were obtained by disregarding damping by plasma electrons, which is valid if the following conditions are fulfilled /167

$$\left(\frac{n_1}{n_0}\right)^{1/3} \frac{v_{Te}}{u_0} \gg 1.$$

If condition (3) is fulfilled, all of the bundle particles are in resonance with the wave, and its hydrodynamic description is possible by means of the moments of the velocity distribution function

$$u^\alpha = \frac{1}{n_\alpha} \int v f_0^\alpha d\vec{v}; \quad T_{ik}^\alpha = \frac{1}{n_\alpha} \int (v_i - u_i^\alpha)(v_k - u_k^\alpha) f_0^\alpha d\vec{v}.$$

We can obtain the equations describing the change in these quantities with time from the kinetic equation of the Fokker Planck type, which is derived in (Ref. 3)

$$\frac{\partial f_0^\beta}{\partial t} = \frac{\partial}{\partial v_i} \left(\alpha_{ik}^\beta \frac{\partial f_0^\beta}{\partial v_k}\right), \tag{7}$$

β = e, i pertain, respectively, to plasma electrons and ions and bundle ions; α_{ik} -- diffusion coefficients in velocity space:

$$\alpha_{\perp\perp} = \frac{e^2}{m_\beta^2} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 \frac{k_\perp^2}{k^2} \sum_{n=1}^{\infty} \frac{n^2 J_n^2 (\lambda_2^\beta)}{\lambda_2^2} [\Gamma_{-n} + \Gamma_n];$$

$$\alpha_{\perp z} = \alpha_{z\perp} = \frac{e^2}{m_\beta^2} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 \frac{k_\perp^2}{k^2} \frac{k_z v_\perp}{\omega_{H\beta}} \sum_{n=1}^{\infty} \frac{n^2 J_n^2 (\lambda_2)}{\lambda_2^2} [\Gamma_{-n} - \Gamma_n]; \quad (8)$$

$$\alpha_{zz} = \frac{e^2}{m_\beta^2} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 \frac{k_z^2}{k^2} \{ J_0^2 (\lambda_2) \Gamma_0 + \sum_{n=1}^{\infty} J_n^2 (\lambda_2) [\Gamma_{-n} + \Gamma_n].$$

Here we have

$$\Gamma_{\pm n} = \frac{\gamma_{\vec{k}}}{\left(\omega_{\vec{k}}^r - k_z v_z \pm n\omega_{Hi}\right)^2 + \gamma_{\vec{k}}^2}; \quad \lambda_2 = \frac{k_\perp v_\perp}{\omega_H}; \quad \omega_{H\beta} = \frac{eH_0}{mc}.$$

The frequency $\omega_{\vec{k}}^r$ and the increment $\gamma_{\vec{k}}$ are determined by the disper- /168 sion relationship of the linear theory (4).

We obtain the system of equations for the change in the "bundle" parameters due to the development of instability from formula (7)

$$\frac{du}{dt} = -\frac{2e^2}{M^2} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 \left\{ \frac{k_z^3}{k^2} \cdot \frac{\gamma_{\vec{k}} \left(k_z u_0 - \omega_{\vec{k}}^r\right)}{\left[\left(k_z u_0 - \omega_{\vec{k}}^r\right)^2 + \gamma_{\vec{k}}^2\right]^2} + \frac{k_\perp^2}{k^2} \cdot \frac{k_z}{4\omega_{Hi}} \times \right.$$

$$\left. \times \left[\frac{\gamma_{\vec{k}}}{\left(k_z u_0 - \omega_{\vec{k}}^r - \omega_{Hi}\right)^2 + \gamma_{\vec{k}}} - \frac{\gamma_{\vec{k}}}{\left(k_z u_0 - \omega_{\vec{k}}^r + \omega_{Hi}\right)^2 + \gamma_{\vec{k}}^2} \right] \right\};$$

$$\frac{dT_{\parallel 1}}{dt} = \frac{2e^2}{M} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 \frac{k_z^2}{k^2} \cdot \frac{\gamma_{\vec{k}}}{\left(k_z u_0 - \omega_{\vec{k}}^r\right)^2 + \gamma_{\vec{k}}^2}; \quad (9)$$

$$\frac{dT_{\perp 1}}{dt} = \frac{2e^2}{M} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 \frac{k_\perp^2}{k^2} \times$$

$$\times \left[\frac{\gamma_{\vec{k}}}{\left(k_z u_0 - \omega_{\vec{k}}^r - \omega_{Hi}\right)^2 + \gamma_{\vec{k}}^2} + \frac{\gamma_{\vec{k}}}{\left(k_z u_0 - \omega_{\vec{k}}^r - \omega_{Hi}\right)^2 + \gamma_{\vec{k}}^2} \right].$$

The change with time of $|E_{\vec{k}}|^2$ is described by the following equation

$$\frac{d\left|E_{\vec{k}}\right|^2}{dt}=2\gamma_{\vec{k}}\left|E_{\vec{k}}\right|^2. \tag{10}$$

We may obtain the equation for the change in the plasma ion parameters from formulas (9), assuming that $u_0^i = 0$. The condition for the applicability of a hydrodynamic description of plasma ions $\frac{k\Delta v_i}{\omega} << 1$ is less hard in the case of $n_1 << n_0$ than equation (3), and this description is applicable throughout the entire development of instability.

Integrating equations (9) with respect to t, we obtain the equations determining the change in the energy of directed bundle motion and in the thermal energy of ions of the bundle and the plasma.

In the case of a weak magnetic field ($\omega_{Hi} << \omega_s$), we have /169

$$\begin{gathered}
n_1 M u_0 \delta u=-\frac{1}{4\pi}\cdot\frac{5}{3}\sum_{\vec{k}}\left|E_{\vec{k}}\right|^2;\\
\frac{1}{2}n_1\delta T_{\parallel 1}\simeq\frac{5^{2/3}}{3}\left(\frac{n_1}{n_0}\right)^{1/3}\left(\frac{v_s}{u_0}\right)^{2/3}\sum_{\vec{k}}\left|E_{\vec{k}}\right|^2\ll\frac{1}{8\pi}\sum_{\vec{k}}\left|E_{\vec{k}}\right|^2;\\
n_1 T_{\perp 1}\simeq\frac{\omega_{0i}^2}{\omega_{Hi}^2}\left(\frac{n_1}{n_0}\right)^{1/3}\left(1-\frac{2}{5}\cdot\frac{v_s^2}{u_0^2}\right)\frac{1}{8\pi}\sum_{\vec{k}}\left|E_{\vec{k}}\right|^2\ll\frac{1}{8\pi}\sum_{\vec{k}}\left|E_{\vec{k}}\right|^2;\\
\frac{1}{2}n_0\delta T_{\parallel i}\simeq\frac{1}{3}\cdot\frac{v_s^2}{u_0^2}\cdot\frac{1}{4\pi}\sum_{\vec{k}}\left|E_{\vec{k}}\right|^2;\\
n_0\delta T_{\perp i}=\frac{5}{3}\left(1-\frac{2}{5}\cdot\frac{v_s^2}{u_0^2}\right)\frac{1}{8\pi}\sum_{\vec{k}}\left|E_{\vec{k}}\right|^2.
\end{gathered} \tag{11}$$

In the opposite case ($\omega_{Hi} >> \omega_s$), we have

$$\begin{gathered}
n_1 M u_0\delta u\simeq-\frac{1}{1-\frac{u_0^2}{v_s^2}}\cdot\frac{1}{4\pi}\sum_{\vec{k}}\left|E_{\vec{k}}\right|^2;\\
n_1\delta T_{\parallel 1}\simeq\left(\frac{n_1}{n_0}\right)^{1/3}\frac{1}{1-\frac{u_0^2}{v_s^2}}\sum_{\vec{k}}\left|E_{\vec{k}}\right|^2;\\
n_0\delta T_{\parallel i}\simeq\frac{1}{1-\frac{u_0^2}{v_s^2}}\cdot\frac{1}{4\pi}\sum_{\vec{k}}\left|E_{\vec{k}}\right|^2.
\end{gathered} \tag{12}$$

The change in the electron distribution function in the case of instability is determined according to the following equation

$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\partial v_z}\left\{\frac{e^2}{m^2} \cdot \frac{1}{(2\pi)^3} \int d\vec{k} \left| E_{\vec{k}} \right|^2 \frac{k_z^2}{k^2} \cdot \frac{\gamma_{\vec{k}}}{\left(\omega_{\vec{k}}^2 - k_z v_z\right)^2 + \gamma_{\vec{k}}^2} \cdot \frac{\partial f_0^e}{\partial v_z}\right\}. \tag{13}$$

The phase velocities of the excited oscillations change between $v_{Ti} \ll v_{\Phi} \ll v_{Te}$. Therefore, the main portion of plasma electrons is in resonance with the waves $v \gg v_{\Phi}$. The change in the electron distribution function for these v is described by the equation /170

$$\frac{\partial f_0^e}{\partial t} = \frac{e^2}{m^2} \cdot \frac{\partial}{\partial v_z}\left\{\sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 \frac{k_z^2}{k^2} \cdot \frac{\gamma_{\vec{k}}}{k_z^2 v_z^2} \cdot \frac{\partial f_0^e}{\partial v_z}\right\}. \tag{14}$$

Thus, for a change in the electron thermal energy in the case of instability, we obtain the following in the case of a weak magnetic field

$$\frac{1}{2} n_0 \delta T_{\parallel e} = \frac{2}{3} \cdot \frac{1}{8\pi} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 \tag{15}$$

and in the case of a strong magnetic field

$$\frac{1}{2} n_0 \delta T_{\parallel e} = \frac{\dfrac{u_0^2}{v_s^2}}{1 - \dfrac{u_0^2}{v_s^2}} \cdot \frac{1}{8\pi} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2. \tag{16}$$

For plasma electrons which are in resonance with excited oscillations, whose velocities lie in the narrow range $\frac{\Delta v}{v_{Te}} \lesssim \sqrt{\frac{m}{M}}$, we obtain the following expression from equation (13)

$$\frac{\partial f_{0e}}{\partial t} = \pi \frac{e^2}{m^2} \cdot \frac{\partial}{dv_z}\left\{\frac{1}{(2\pi)^3} \int d\vec{k} \left| E_{\vec{k}} \right|^2 \frac{k_z^2}{k^2} \delta\left(\omega_{\vec{k}} - k_z v_z\right) \frac{\partial f_{0e}}{\partial v_z}\right\}. \tag{17}$$

Changing to the variable $\xi = \frac{v}{v_s}\left(\sqrt{\frac{T_i}{T_e}} < \xi < 1\right)$ in formula (17) and representing f_{0e} in the following form

$$f_{0e} = \frac{1}{\sqrt{2\pi} v_{Te}} \left(1 - \frac{\varphi(\xi, t) v_s^2}{v_{Te}^2}\right) f_e\left(\vec{v}_\perp\right), \tag{18}$$

where $\varphi\ (\xi,\ t)|_{t=0} = \frac{\xi^2}{2}$, $f_e\left(\vec{v}_\perp\right)$ is the portion of the electron distribution function depending on $v_\perp$, we obtain the following equation for the change in $\phi(\xi,\ t)$

$$\frac{\partial \varphi}{\partial t} = \frac{1}{2\pi} \cdot \frac{e^2}{m^2} \cdot \frac{\omega_k^2}{v_s^5} \cdot \frac{\partial}{\partial \xi} \left\{ \int_0^{\pi/2} \left| E_{\vec{k}} \right|^2 \frac{\sin \theta}{\cos \theta} d\theta \frac{1}{\xi^3} \cdot \frac{\partial \varphi}{\partial \xi} \right\}. \tag{19}$$

The interaction of resonance electrons with oscillations leads to the occurrence of a plateau in the electron distribution function in the region of excited oscillation phase velocities. The time required to establish the plateau may be determined from formula (19): /171

$$\tau \sim \frac{(\Delta\xi)^2}{D};\ D = \frac{1}{2\pi} \cdot \frac{e^2}{m^2} \cdot \frac{\omega_K^2}{v_s^5} \int_0^{\pi/2} \left| E_{\vec{k}} \right|^2 \frac{\sin \theta}{\cos \theta} d\theta \tag{20}$$

($\Delta\xi$ -- the dimensionless width of the plateau).

The change in the energy of resonance electrons is

$$\frac{1}{2} n_0 \delta T_{\parallel e} \sim \frac{\omega_{0i}^2}{\omega_s^2} \left(\frac{n_0}{n_1}\right)^{1/3} \frac{u_0^2}{\frac{T_e}{M_i}} \cdot \frac{u_0}{\left(\frac{T_e}{m}\right)^{1/2}} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 \ll \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2, \tag{21}$$

i.e., it is considerably less than the change in the total energy of plasma electrons.

We may determine the balance of energy during the development of instability from formulas (11) and (15), (12) and (16): The energy of directed motion, which is lost by a bundle, changes into the thermal energy of particles in the bundle and the plasma and into energy of electrostatic oscillations. Since conditions (3) must be fulfilled in the initial (hydrodynamic) stage, we may readily estimate the maximum energy of oscillations excited at this stage:

$$\frac{1}{8\pi} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 \lesssim \left(\frac{n_1}{n_0}\right)^{1/3} n_1 M u_0^2. \tag{22}$$

At this oscillation energy, the time required to establish a plateau in the electron distribution function, as follows from formula (20), is

$$\tau \sim \left(\frac{n_0}{n_1}\right)^{1/3} \left(\frac{m}{M}\right)^2 \frac{1}{\omega}, \tag{23}$$

i.e., it is a little less than the time required for the development of instability in the initial, quasilinear stages.

Thus, the plasma electron distribution function changes most rapidly in the resonance region: A plateau appears in this region in the velocity range in which oscillations are excited at a given moment of time. A change in the spectral density of oscillations and ion parameters occurs much more slowly. Under these conditions, the electrons have no significant influence on the dynamics of the instability development.

Excitation of Low Frequency Oscillations (Quasilinear Stage) /172

Further development of instability leads to a still greater increase in the thermal scatter in the bundle, and its distribution function becomes so diffused that the quasilinear approximation is applicable.

In the case of $\omega_s \gg \omega_{Hi}$, it follows from the expressions for the diffusion coefficients (8) that $\alpha_{\perp\perp} \sim \alpha_{zz}$, if $k\delta v \sim \omega$. The problem is thus essentially a three-dimensional problem. Let us investigate the opposite case, $\omega_{Hi} \gg \omega_s$, since in this case $\alpha_{\perp\perp} \ll \alpha_{zz}$ at the quasilinear stage, i.e., only longitudinal diffusion is significant.

The initial system of equations for the quasilinear approximation has the following form

$$\frac{\partial g}{\partial t} = \pi \frac{\partial}{\partial v} \left[\frac{e^2}{M^2} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 \frac{k_z^2}{k^2} \delta \left(\omega^r_{\vec{k}} - k_z v_z \right) \frac{\partial g}{\partial v} \right]; \tag{24}$$

$$\frac{\partial \left| E_{\vec{k}} \right|^2}{\partial t} = \pi \omega^r_{\vec{k}} \frac{n_1}{n_0} \cdot \frac{\omega_s^2}{k^2} \frac{\partial g}{\partial v} \bigg|_{v = \frac{\omega^r_{\vec{k}}}{k_z}} \left| E_{\vec{k}} \right|^2. \tag{25}$$

Here $g(t, v)$ is the distribution function of bundle ions integrated with respect to $\vec{v}_\perp$.

Substituting $\frac{\partial g}{\partial v} \left| E_{\vec{k}} \right|^2$ from equation (25) in formula (24), changing from summation over $\vec{k}$ to integration, and integrating with respect to k due to the δ-function, we obtain

$$\frac{\partial g}{\partial t}=\frac{1}{2\pi^2}\cdot\frac{n_0}{n_1}\cdot\frac{e^2}{M^2}\cdot\frac{\partial}{\partial v}\left\{\int_0^{\pi/2}\sin\theta d\theta\frac{\omega_{0l}}{v^5\left(1-\frac{v^2}{v_s^2}\right)^{1/2}}\cdot\frac{\partial\left|E_{\vec{k}}\right|^2}{\partial t}\right\}. \tag{26}$$

Integrating with respect to v and t, we may write the following equation

$$\int_0^{\pi/2}\sin\theta d\theta\left\{\left|E_{\vec{k}}(t)\right|^2-\left|E_{\vec{k}}(0)\right|^2\right\}=2\pi^2\frac{M^2}{e^2}\cdot\frac{n_1}{n_0}\cdot\frac{1}{\omega_{0l}}\times$$
$$\times v^5\left(1-\frac{v^2}{v_s^2}\right)^{1/2}\int_{v_1}^{v}[g(t,v')-g(0,v')]\,dv', \tag{27}$$

where v_1 is the lower boundary of the instability region caused by damping /173 by plasma ions. We may employ this equation to determine the oscillation energy at the quasilinear stage:

$$W=\frac{1}{8\pi}\sum_{\vec{k}}\left|E_{\vec{k}}(\infty)\right|^2=\frac{1}{2}Mn_1\int_{v_1}^{v_2}v\left(1-\frac{v^2}{v_s^2}\right)\int_{v_1}^{v}[g(\infty,v')-$$
$$-g(0,v')]\,dv'dv, \tag{28}$$

where $g(\infty, v)$ is the distribution function at the end of the quasilinear stage -- the plateau, whose height is determined from the following condition

$$g(\infty,v)(v_2-v_1)=n_1;\ g(\infty,v)=\frac{n_1}{v_2-v_1}; \tag{29}$$

v_1 and v_2 (the upper boundary of the instability region) are determined by the following relationships

$$g(\infty,v_1)=g_i(v_1);\ g(\infty,v_2)=g_0(v_2); \tag{30}$$

$g_0(v)$ -- the distribution function of the ion bundle at the beginning of the quasilinear stage; $g_i(v)$ -- the distribution function of plasma ions integrated with respect to $\vec{v}_\perp$. We obtain the following from (30)

$$v_1=\sqrt{\frac{2T_i}{M}}\ln^{1/2}\left[\frac{n_0u_0}{n_1\left(\frac{2\pi T_i}{M}\right)^{1/2}}\right]\ll u_0;\ v_2\simeq u_0\left(1-\left(\frac{n_1}{n_0}\right)^{1/3}\right). \tag{31}$$

Performing integration with respect to v in (28), we may write

$$W = \frac{1}{6} M n_1 u_0^2 \left(1 - \frac{3}{5} \cdot \frac{u_0^2}{v_s^2}\right). \tag{32}$$

The change in the energy of the plasma ion thermal motion may be determined from (11):

$$\frac{1}{2} n_0 \delta T_{\parallel i} = \omega_{0i}^2 \frac{1}{2\pi} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 \frac{k_z^2}{k^2} \cdot \frac{1}{\omega_{\vec{k}}^{r2}} = \frac{1}{6} M n_1 v_2^2. \tag{33}$$

The increase in the energy of the plasma electron thermal motion is

$$\frac{1}{2} n_0 \delta T_{\parallel e} \simeq \frac{1}{8\pi} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 \frac{1}{k^2 \lambda_{De}^2} = \frac{1}{10} M n_1 \frac{v_2^4}{v_s^2}. \tag{34}$$

The finite bundle velocity established at the end of the quasilinear stage is /174

$$u^\infty = \int_{v_1}^{v_2} v g(\infty, v)\, dv = \frac{v_2 - v_1}{2} \simeq \frac{u_0}{2},$$

i.e., the energy loss of the ordered bundle motion

$$\delta\varepsilon = -\frac{3}{8} n_1 M u_0^2. \tag{35}$$

The thermal bundle energy acquired during the development of instability is

$$\frac{n_1 \delta T_{\parallel 1}}{2} = \int_{v_1}^{v_2} g(\infty, v) \left(v - \frac{v_2 + v_1}{2} \right)^2 \frac{m}{2}\, dv \simeq \frac{n_1 M u_0^2}{24}. \tag{36}$$

We may employ formulas (32) - (36) to verify the fact that the law of conservation of energy is fulfilled

$$\delta\varepsilon + \frac{1}{8\pi} \sum_{\vec{k}} \left| E_{\vec{k}} \right|^2 + \frac{1}{2} n_0 \delta T_{\parallel i} + \frac{1}{2} n_0 \delta T_{\parallel e} + \frac{1}{2} n_1 \delta T_{\parallel 1} = 0. \tag{37}$$

Thus, in contrast to the excitation of high frequency oscillations, the excitation of low frequency oscillations leads to the transfer of a considerable portion $\left(\sim\frac{1}{3}\right)$ of the bundle energy to plasma ions, and also leads to significant heating of the ion component.

The quasilinear theory disregards the nonlinear phenomena of oscillation scattering by plasma particles. As was shown in (Ref. 1), in the case of the excitation of Langmuir oscillations by an electron bundle, nonlinear interaction of harmonics is usually insignificant in the quasilinear stage. However, there is a parameter region in which these phenomena determined the dynamics of the instability development (Ref. 4).

We shall continue to study the role of the nonlinear interaction of harmonics in the case (which we are considering) of excitation of ion-sound oscillations by an ion bundle.

Excitation of Shortwave, Low Frequency Oscillations (Hydrodynamic Stage) /175

When allowance is made for the finite, Larmor radius of plasma ions, excitation of longitudinal oscillations by the harmonics $\sim n\omega_{Hi}$ is possible. One important feature of these oscillations is excitation in the case of $T_i \sim T_e$ and propagation almost perpendicularly to the magnetic field. Therefore, it is more likely that low frequency oscillations are the reason for anomalous plasma diffusion perpendicularly to the magnetic field, rather than ion-sound oscillations, for which $\frac{k_\perp}{k_z} << 1$.

Let us investigate the manner in which the ion bundle and the plasma parameters change at the initial excitation stage of low frequency oscillations.

The dispersion equation for shortwave oscillations for conditions (3) has the following form

$$1+\frac{T_i}{T_e}-e^{-\mu_i}\sum_{n=-\infty}^{\infty}\frac{\omega I_n(\mu_i)}{\omega-n\omega_{Hi}}-\frac{\omega_{01}^2}{(\omega-k_z u_0)^2}\cos^2\theta-\frac{\omega_{01}^2}{(\omega-k_z u_0)^2-\omega_{Hi}^2}\times$$
$$\times\sin^2\theta=0. \quad (38)$$

Waves with frequencies of $\omega_n = n\omega_{Hi}(1+\psi_n)$ have the largest increment. Their increasing increment in the case of n = 1 is

$$\gamma_{\vec{k}}=\frac{\sqrt{3}}{2}\left(\frac{n_1}{n_0}\cdot\frac{T_i}{Mu_0^2}\alpha(\vec{k})\right)^{1/3}\omega_{Hi},$$

where

$$\alpha(\vec{k})=\frac{\psi_1^2 e^{\mu_i}}{I_1(\mu_i)}.$$

The change in the ordered velocity and thermal energy of bundle ions may be described by means of (9), where $\omega_{\vec{k}}$ and $\gamma_{\vec{k}}$ are determined from formulas (38) - (39). Making use of the fact that $\gamma_{\vec{k}}$ has a maximum with respect to $\vec{k}$, we may obtain the following in the case of instability at the frequency $\omega = \omega_{Hi}(1 + \psi_1)$

$$n_1 M u_0 \delta u = -\left(\frac{\omega_{0i}^2}{k^2 \frac{T_i}{M}} \cdot \frac{1}{\alpha(\vec{k})}\right)_{\vec{k}=\vec{k}_0} \frac{1}{8\pi} \cdot \frac{1}{(2\pi)^3} \int d\vec{k} \left|E_{\vec{k}}\right|^2;$$

$$n_1 \delta T_{\perp 1} = \frac{n_1}{n_0} \cdot \frac{\omega_{0i}^2}{\omega_{Hi}^2} \left(\frac{k_\perp^2}{k^2}\right)_{\vec{k}=\vec{k}_0} \frac{1}{8\pi} \cdot \frac{1}{(2\pi)^3} \int d\vec{k} \left|E_{\vec{k}}\right|^2; \tag{40}$$

$$\frac{1}{2} n_1 \delta T_{\parallel 1} = \frac{\omega_{0i}^2}{\omega_{Hi}^2} \left[\frac{k_z^2 u_0^2}{k^2 \frac{T_i}{M}} \left(\frac{n_1}{n_0} \cdot \frac{T_i}{M u_0^2}\right)^{1/3} \frac{1}{\alpha^{2/3}(\vec{k})}\right]_{\vec{k}=\vec{k}_0} \frac{1}{8\pi} \cdot \frac{1}{(2\pi)^3} \int \left|E_{\vec{k}}\right|^2 d\vec{k}$$

($\vec{k}_0$ -- the value of $\vec{k}$ at which $\gamma_{\vec{k}}$ has a maximum). Let us employ the approximate formulas given in (Ref. 2), and we shall assume that $\alpha(\vec{k})$ reaches a maximum in the case of $\frac{k_{\perp 0}^2 v_{Ti}^2}{\omega_{Hi}^2} \simeq 1.5$: /176

$$\alpha(\vec{k}) \simeq \frac{0{,}22}{\left(1 + \frac{T_i}{T_e}\right)^2} \ll 1; \quad \frac{k_{\parallel 0}}{k_{\perp 0}} \sim \frac{v_{Ti}}{u_0}. \tag{41}$$

Assuming that $\frac{v_{ri}}{u_0} \ll \psi_1 \ll 1$, we obtain the following expression for the plasma ion diffusion coefficients in the case of instability at the frequency $\omega = \omega_{Hi}(1 + \psi_1)$

$$\alpha_{zz} = \frac{e^2}{M^2} \cdot \frac{1}{(2\pi)^3} \int d\vec{k} \left|E_{\vec{k}}\right|^2 \frac{k_z^2}{k^2} \frac{\gamma_{\vec{k}} J_1^2\left(\frac{k_\perp v_\perp}{\omega_{Hi}}\right)}{\left(\omega_{\vec{k}}^r - \omega_{Hi}\right)^2 + \gamma_{\vec{k}}^2};$$

$$\alpha_{\perp\perp} = \frac{e^2}{M^2} \cdot \frac{1}{(2\pi)^3} \int d\vec{k} \left|E_{\vec{k}}\right|^2 \frac{k_\perp^2}{k^2} \cdot \sum_{n=1}^{\infty} \frac{J_1^2(\lambda_2^i)}{\lambda_2^{i2}} \times$$
$$\times \frac{\gamma_{\vec{k}}}{\left(\omega_{\vec{k}}^r - \omega_{Hi}\right)^2 + \gamma_{\vec{k}}^2}; \tag{42}$$

$$\alpha_{\perp z}=\frac{e^2}{M^2}\cdot\frac{1}{(2\pi)^3}\int d\vec{k}\left|E_{\vec{k}}\right|^2\frac{k_\perp^2}{k^2}\cdot\frac{k_z v_\perp}{\omega_{Hi}}\sum_{n=1}^{\infty}\frac{J_1^2\left(\lambda_2^i\right)}{\lambda_2^{i2}}\times$$

$$\times\frac{\lambda_{\vec{k}}}{\left(\omega^r_{\vec{k}}-\omega_{Hi}\right)^2+\gamma^2_{\vec{k}}}.$$

We obtain the following by means of (42) from formula (7): /177.

$$\frac{1}{2}n_0\delta T_{\parallel i}=\frac{\omega_{0i}^2}{\omega_{Hi}^2}\left(\frac{k_z^2}{k^2}\cdot\frac{1}{\alpha\left(\vec{k}\right)}\right)_{\vec{k}=\vec{k}_0}\frac{1}{8\pi}\cdot\frac{1}{(2\pi)^3}\int d\vec{k}\left|E_{\vec{k}}\right|^2;$$

$$n_0\delta T_{\perp i}=\left(\frac{\omega_{0i}^2}{k^2\frac{T_i}{M}}\cdot\frac{1}{\alpha\left(\vec{k}\right)}\right)_{\vec{k}=\vec{k}_0}\frac{1}{8\pi}\cdot\frac{1}{(2\pi)^3}\int d\vec{k}\left|E_{\vec{k}}\right|^2. \tag{43}$$

We may determine the change in the plasma electron energy by formula (14)

$$\frac{1}{2}n_0\delta T_{\parallel e}=\left(\frac{\omega_{0e}^2}{k^2\frac{T_e}{m}}\right)_{\vec{k}=\vec{k}_0}\frac{1}{8\pi}\cdot\frac{1}{(2\pi)^3}\int d\vec{k}\left|E_{\vec{k}}\right|^2. \tag{44}$$

Comparing expressions (40), (43) and (44), we can see that the energy of the bundle ordered motion changes primarily into energy of the transverse thermal motion of plasma ions, and may lead to significant ion diffusion perpendicularly to the magnetic field. Relationships (40), (43) and (44) are valid as long as the conditions of a monoenergetic bundle are fulfilled (3).

The maximum energy of low frequency fields, obtained at this stage, is

$$\frac{1}{8\pi}\sum_{\vec{k}}\left|E_{\vec{k}}\right|^2\sim\frac{\omega_{Hi}^2}{\omega_{0i}^2}\,\alpha\left(\vec{k}\right)\left(\frac{n_1}{n_0}\cdot\frac{T_i}{Mu_0^2}\,\alpha\left(\vec{k}\right)\right)^{1/3}n_1Mu_0^2. \tag{45}$$

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NONLINEAR PHENOMENA IN A PLASMA WAVE GUIDE (ION CYCLOTRON RESONANCE AT A DIFFERENCE FREQUENCY)

B. I. Ivanov

Ion cyclotron resonance (ICR) has been extensively studied both theoretically and experimentally [see, for example, the summary in (Ref. 1)]. Several works have appeared recently (Ref. 2 - 4) which examined the problems of the nonlinear theory of ion and electron cyclotron resonance. As is known, the non-linearity criterion has the following form (Ref. 5, 6) /178

$$\frac{eE_0\lambda}{2\pi mc^2\beta_\Phi}\left(1-\frac{v_0}{v_\Phi}\right)^{-1}\sim 1$$

(E_0 -- strength of the wave field; λ -- wave length; $\beta_\Phi \frac{v_\Phi}{c}$ -- retardation; v_0 -- ordered plasma velocity). Formation of nonlinear phenomena is facilitated during resonance (Ref. 6), since in this case the non-linearity parameter contains the additional factor $\sim \omega\ (\omega - \omega_c)^{-1}$. Thus, it is possible that nonlinear phenomena may occur in the case of ICR, because for this case the occurrence of large strengths of the wave field, small phase velocities, and large wavelengths is characteristic. In this case, nonlinear phenomena may play a significant role during heating (nonlinear damping, nonlinear shift of the resonance frequency) and during the introduction of high frequency energy into the plasma (interaction of frequencies).

In principle, it is possible to introduce large UHF power into the plasma at two frequencies, and then to perform ICR at the difference frequency. Such a mechanism is also possible during the excitation of low frequency oscillations in the plasma-bundle system (Ref. 7). As is known, in unstable plasmas, low frequency oscillations, whose origin is sometimes difficult to establish, occur simultaneously with high frequency oscillations. The occurrence of the high frequency oscillations is satisfactorily explained by the theory of plasma-bundle interaction.

One of the mechanisms leading to the formation of low frequency oscillations may be the nonlinear interaction of high frequency oscillations [separation of difference frequencies, decay instabilities (Ref. 8, 9)] with the subsequent transfer of energy from high frequency oscillations to low frequency oscillations [parametric amplification (Ref. 10)]. This article makes an attempt to provide a model for this mechanism by /179
which low frequency oscillations are excited close to the ion cyclotron frequency. The parameters of the apparatus are the same as in the preceding article (Ref. 12), which investigated the nonlinear distortions of the signal form and the formation of combined frequencies. We employed generators having a small power ($\sim$1 w) which had a relatively small disturbing influence on the plasma which was produced independently. In order to fulfill the non-linearity condition, it was necessary to operate at low frequencies ($f \sim 1$ Mc) and with low phase velocities ($\beta_\Phi \sim 10^{-2}$), which, in its turn, made it necessary to employ a low-density plasma ($n \sim 10^9$ cm^{-3}) (Ref. 12). On the other hand, $f_- \gtrsim f_{ci} >> \nu_{in}$ ($\nu_{in} \sim 10^8$ p -- collision frequency) represents the necessary condition for observing the ICR at the difference frequency. In view of these considerations, the main ("beat") frequencies and the difference frequency were of order of magnitude one: $f_1 \sim f_2 \sim f_- \sim 1$ Mc.

In order to observe the weak ICR signal, a sensitive system of a balanced, high frequency bridge was employed (measures were taken to reduce the noise level). Figure 1 shows the diagram of the apparatus (1 -- /180
current regulator; 2 -- voltage regulator; 3 - high frequency generators; 4 -- amplifiers; 5 -- phase inverter; 6 -- phase rotators; 7 -- AVC unit; 8 -- two-ray oscillograph; 9 -- heterodyne receiver; 10 -- heterodyne frequency meter; 11 -- self-excited oscillator; 12 -- main anode; 13 -- quartz tube; 14 -- water cone; 15 -- auxilliary anode; 16 -- cathode; 17 -- magnetic field recorder; 18 -- palladium filters). The plasma wave guide consisted of the following, which were distributed coaxially: A plasma core with a diameter of 1 cm, a lead tube with a diameter of 3 cm, a copper casing with a diameter of 23 cm with the cross section along the generatrix, and a solenoid. The total length was about 180 cm. The quartz discharge tube was evacuated from both sides to a vacuum of $\sim 10^{-4}$ n/m^2, after which hydrogen was introduced from both sides through the palladium filters. When the entire length of the tube was continuously evacuated, it was possible to obtain a constant pressure. Before the measurements, the tube was treated to preliminary processing with prolonged, high frequency discharge (wavelength $\lambda \cong 2$ m, generator power $P \sim 500$ w). The plasma was produced by discharge at a constant current, and the discharge current was stabilized. In order to increase the

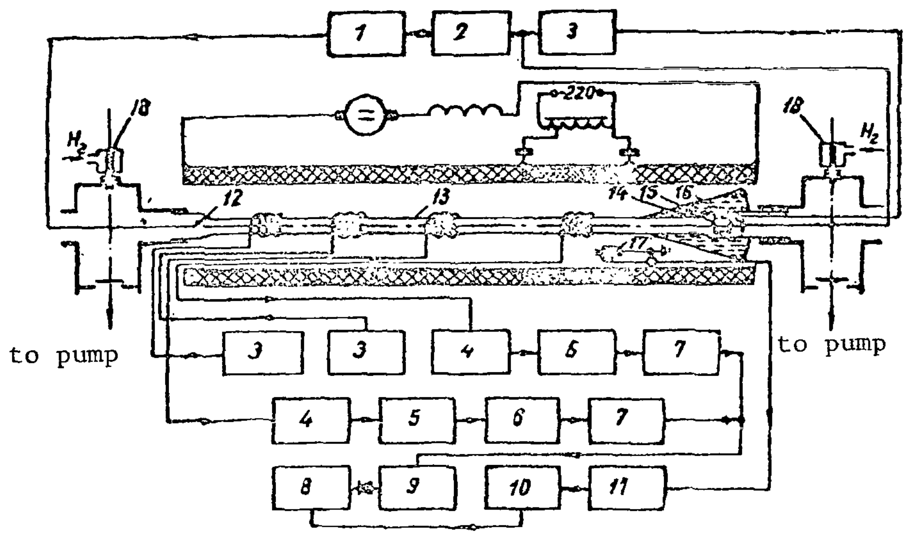

Figure 1

ɔnization coefficient, a cathode was employed made of lanthanum hexaɔride with indirect direct current heating. Two anodes were emloyed in order to obtain a stable discharge: The main anode and the uxilliary anode. The anode potentials were selected according to the lasma noise minimum.

The main frequencies f_1 and f_2 from the generators were excited in ne plasma wave guide by short spirals located close to the left end of he wave guide. An adiabatic, absorbing water charge (length of about 0 cm) was located at the right end of the wave guide. The reflection oefficient from the right end of the wave guide k equalled 0.1 - 0.3 Ref. 13). Thus, moving waves with main (f_1 and f_2) and combined nf$_1 \pm mf_2$; n and m -- whole numbers) frequencies could be propagated n the wave guide. The output signal was employed on two spirals, one f which was located in the magnetic field section which could be odulated by a commercial frequency. From the receiving spirals, the ignal was supplied to the two arms of the high frequency bridge. Each rm consisted of an amplifier, a phase rotator, and the AVC unit. The atter was used to eliminate relatively slow unbalancing of the bridge $\tau_{AVC} >> T_{mod}$). The signals from the arm of the bridge were supplied ut of phase to the heterodyne receiver adjusted to the resonance freuency, and after rectification through the low-frequency filter they ere supplied to the oscillograph. In the normal position, both arms ere almost completely balanced, but the amplitude of the signal with a odulating frequency remained somewhat larger. During the modulation /181

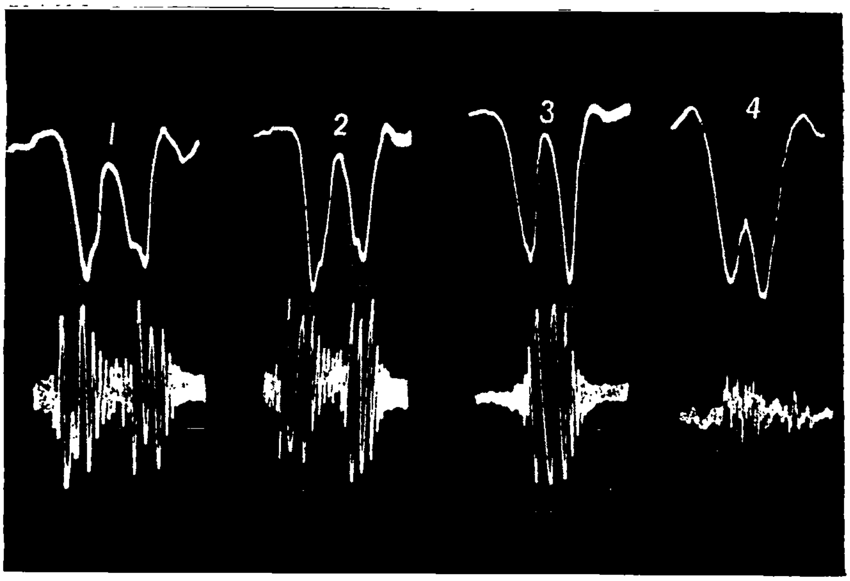

Figure 2

of the magnetic field, at the moment the resonance value was crossed ($f_- \simeq f_{ci}$) -- twice during the modulation period -- a signal for the bridge unbalance was produced, which could be observed on the oscillograph. This recording system has good sensitivity and noiseproof qualities (subtraction of the signals from the two receiving channels leads to an increase in the modulation depth of the carrier frequency by the ICR signal, and simultaneously eliminates the plasma noise correlations along the wave guide length).

Figure 2 presents oscillograms showing the dependence of the resonance position on the magnetic field strength (f_1 = 1.5 Mc, f_2 = 2.5 Mc, f_- = 1.0 Mc; $n = 1 \cdot 10^9$ cm^{-3}, $p = 7 \cdot 10^{-2}$ n/m^2, $\tilde{H}_\alpha$ = 7 ka/m, $H \simeq$ 56-62 ka/m). The upper line corresponds to the resonance signal which is inverted during rectification. The lower line corresponds to the signal coming from the generator recorder of the magnetic field. With an increase in /182
the constant magnetic field strength (for a fixed difference frequency and a constant amplitude of the variable magnetic field), the resonances converge, since the resonance condition ($f_- \simeq j_{ci}$) is fulfilled in the negative halfperiod of the variable magnetic field.

Figure 3 shows the dependence of the resonance position on plasma density (f_1 = 1.5 Mc, f_2 = 2.5 Mc, f_- = 1.0 Mc, H = 62 ka/m, $\tilde{H}_\alpha$ = 7 ka/m,

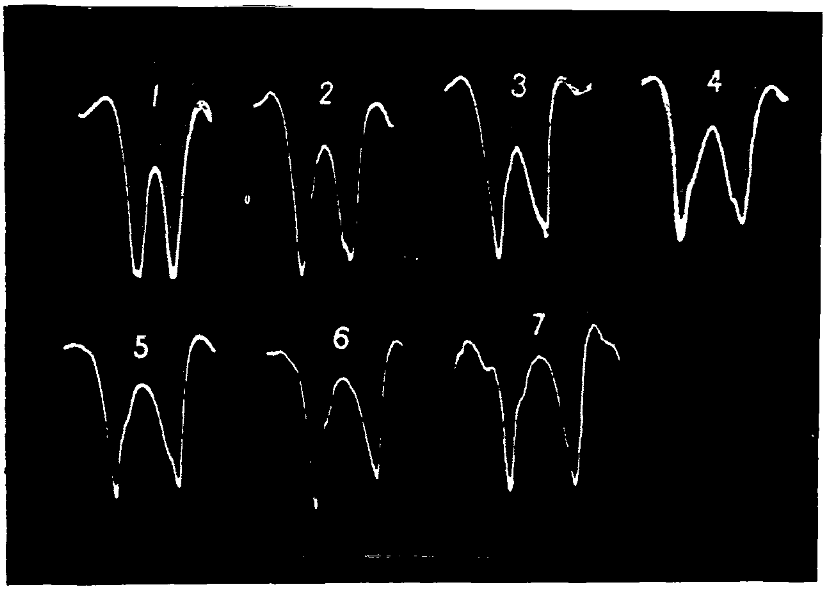

Figure 3

$p = 7 \cdot 10^{-2}$ n/m^2, $n \simeq (1 - 2)\ 10^9$ cm^{-3}). With an increase in the plasma density, the resonances diverge, i.e., they shift into the region of large magnetic field strengths.

In order to obtain quantitative estimates, the reflection coefficients from the wave guide ends, the magnetic field strength, the phase velocity, and the plasma density were measured.

Dynamic measurements of the magnetic field strength were performed by the generator recorder (Ref. 11) (see Figure 1). The self-excited oscillator circuit was located in the modulated section of the solenoid. Carbonyl iron was used as the induction core. Due to the small dimensions and the small value of μ (~ 10), the recorder disturbed the magnetic field to an insignificant extent. With a change in the magnetic field /183 strength, due to the dependence $\mu(H)$ the circuit inductance and the self-excited oscillator frequency changed. The latter was measured by the heterodyne frequency meter according to the zero beats, which could be recorded simultaneously with the ICR signal by the two-ray oscillograph (see Figure 2). This system was calibrated initially by nuclear magnetic resonance.

The phase velocity was measured by the system shown in Figure 4 (1 -- AVC unit; 2 -- phase meter; 3 -- amplifier; 4 -- delay line;

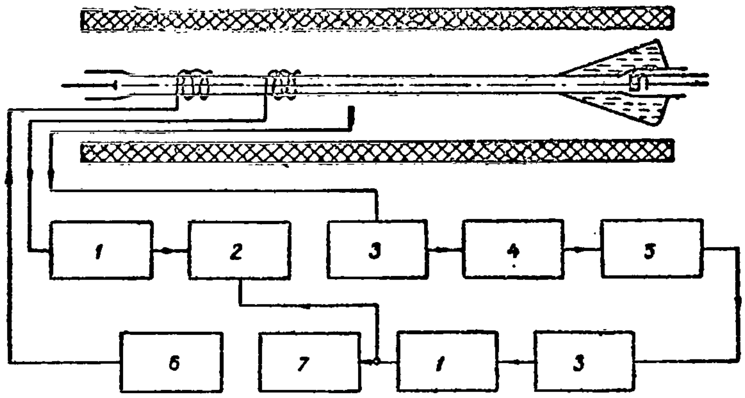

Figure 4

5 -- limiting attenuator; 6 -- generator; 7 -- selective microvoltmeter). A wave with the frequency $f = f_- \simeq f_{ci}$ was excited in the plasma wave guide by the generator. A high frequency signal was applied to the two antennae (a non-mobile short spiral and a mobile whip), and was supplied to the phase meter input. The compensation method was employed to measure β_Φ. A coaxial delay line, consisting of segments of the high frequency cable (length, 2.5; 5; 10 and 20 m), was switched into the circuit of the mobile collapsible-whip antenna. These high frequency cable segments could be subsequently combined in any combinations. With a minimum distance between the antennae, the delay line was completely introduced, and the phase meter indicator pointed to zero. As the distance between the antennae increased, the delay line decreased so as to compensate for the phase shift produced. In spite of the fact that the /184
phase meter system was designed so that the phase reading was not dependent on the signal amplitude, there was a possibility of error for small phase shifts and significant changes in the signal amplitude. In order to eliminate this possibility, an AVC unit was switched into the circuit of both antennae, and also a selective microvoltmeter (for controlling the high frequency signal amplitude) and a limiting attenuator (for maintaining the amplitude at a definite level) were introduced into the circuit of the mobile whip.

The length of the delay line ΔL changed linearly as a function of the distance between the antennae ℓ. The retardation was determined according to the following relationship

$$\beta_\Phi = \frac{l}{\Delta L \sqrt{\varepsilon}} \approx 3 \cdot 10^{-2}$$

(ε -- dielectric constant of cable insulation).

The electron density n was measured by the shift in the eigen resonator frequency Δf, and the dependence $\Delta f(n)$ was determined by preliminary calibration with respect to the electron bundle (Ref. 12). The relative content of atomic and molecular hydrogen ions in the plasma was not measured.

The following conclusions may be drawn on the basis of the measurements performed. At the moment that resonance is passed, the amplitude of the "difference" wave increased. The resonance frequency increased with an increase in the magnetic field strength, and decreased with an increase in the plasma density, while $f_- \simeq f_{ci}$. The picture observed corresponds qualitatively to the excitation of ion cyclotron waves at the difference frequency.

The experimental data agree qualitatively with the dispersion relationship for ion cyclotron waves

$$\omega^2 = \omega_{ci}^2\left(1 - \beta_\Phi^2 \frac{\omega_i}{\omega_{ci}^2}\right).$$

The quantitative divergences (stronger experimental dependence of resonance frequency on plasma density) do not as yet yield to a satisfactory explanation.

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SECTION IV

EXCITATION OF PLASMA OSCILLATIONS

RADIATION OF ELECTRONS IN THE PLASMA-MAGNETIC FIELD BOUNDARY LAYER /186

V. V. Dolgopolov, V. I. Pakhomov, K. N. Stepanov

The cyclotron radiation of electrons in the plasma-magnetic field boundary layer can make a significant contribution to the energy balance of thermonuclear reactors with a small density, which employ magnetic grids to contain the plasma. This problem was examined in (Ref. 1).

The thickness of the transitional layer between the plasma and the magnetic field may comprise several Larmor electron radii

$$\rho_e = \frac{v_e}{\omega_B}\left(v_e = \sqrt{\frac{T}{m}},\ \omega_B = \frac{eB_0}{mc}\right).$$

The trajectory curvature of electrons having a velocity on the order of v_e is also on the order of ρ_e. Moving along such a trajectory in a vacuum, a non-relativistic electron radiates the following energy per unit of time /187

$$\frac{dw}{dt} \sim \frac{e^2\omega_B^2 v_e^2}{c^3},\ T \ll mc^2. \quad (1)$$

The number of emissive electrons per unit of layer area is $n_0\rho_e$ (n_0 -- plasma density). Therefore, the total intensity of cyclotron radiation from unit of layer surface -- if it is assumed that all the electrons in the layer radiate the same way as in a vacuum -- is

$$I \sim \frac{dw}{dt} n_0\rho_e \sim \frac{e^2 n_0 \omega_B v_e^3}{c^3}. \quad (2)$$

Since

$$B_0^2 \sim 8\pi n_0 T, \quad (3)$$

$$I = \alpha \frac{e^3 n_0^{3/2} T^2}{c^4 m^{5/2}}, \quad (4)$$

where $\alpha \sim 1$. Expression (4) coincides, within an accuracy of a coefficient on the order of unity, with the result derived by Burhardt (Ref. 1).

We may employ expression (1) only if the refractive index n for radiated frequencies $\omega \sim \omega_B$ is close to unity. In the case under consideration, the radiated frequencies lie in the region of the anomalous skin-effect

$$\mathrm{Re}\, n \sim \mathrm{Im}\, n \sim \frac{\Omega}{\omega_B} \sim \frac{c}{v_e} \gg 1 \tag{5}$$

$\left(\Omega = \sqrt{\frac{4\pi e^2 n_0}{m}}\right.$ -- plasma frequency$\left.\right)$. In a medium with a large, complex refractive index (5) the radiation intensity differs greatly from the radiation intensity (1). For example, in a dense, non-relativistic plasma $\left(\Omega^2 \gtrsim \frac{v_e}{c}\omega_B^2\right)$, the intensity of cyclotron radiation of the main frequency decreases, as compared with radiation in a vacuum, by a factor of $\frac{mc^2}{T}$ (Ref. 2 - 4).

Let us determine the intensity of cyclotron radiation, assuming that the radiated waves correspond to stable plasma oscillations. In the case of radiation equilibrium, the radiation flux falling on the plasma-magnetic field boundary, $I_{RJ} = \frac{\omega^2 T}{4\pi^3 c^2}$, equals the sum of the flux emanating from the plasma $I(\omega)$ and the flux reflected from the plasma $I_{RJ}R$ (R -- reflection coefficient). We thus find that $I(\omega) = I_{RJ}(1 - R)$. In the case under /188 consideration, $1 - R \sim \frac{1}{n} \sim \frac{v_e}{c}$. The width of the radiation spectrum, caused by the Doppler effect during radiation and by the nonuniformity of the magnetic field in the boundary layer, is on the order of ω_B in the case under consideration. Therefore, the total intensity of electron cyclotron radiation in the layer is

$$I \sim I(\omega)\,\omega_B \sim \frac{\omega_B^3 v_e T}{4\pi^3 c^3} \sim \frac{e^3 n_0^{3/2} T^3}{c^6 m^{7/2}}. \tag{6}$$

A comparison of (6) and (4) shows that allowance for plasma polarization decreases the intensity of cyclotron radiation by a factor of $\frac{mc^2}{T}$.

Due to plasma resonance, radiation of boundary layer electrons strongly increases in the region of frequencies ω which are less than the maximum Langmuir frequency Ω_0, but considerably greater than ω_B. In the resonance region where $\Omega(x) = \sqrt{\frac{4\pi e^2 n_0(x)}{m}} \sim \omega$, the intensity of electron braking radiation greatly increases at the frequency ω.

Let us investigate absorption of waves whose electric vector lies in the plane of incidence (the XY plane), and who impact on the plasma layer from a vacuum. The electric field component which is parallel to the plasma boundary, $E_y \sim e^{ik_y y - i\omega t}$, satisfies the following equation

$$\frac{d^2E_y}{dx^2} - \frac{k_y^2c^2}{\omega^2\varepsilon\left(\varepsilon - \frac{k_y^2c^2}{\omega^2}\right)} \cdot \frac{d\varepsilon}{dx} \cdot \frac{dE_y}{dx} + \left(\frac{\omega^2}{c^2}\varepsilon - k_y^2\right)E_y = 0, \tag{7}$$

where $\varepsilon = 1 - \frac{\Omega^2(x)}{\omega^2} + i\frac{\nu(x)\,\Omega^2(x)}{\omega^3}$ is the dielectric constant of the plasma;

$\nu(x) = \frac{\sqrt{\pi}e^4 n_0(x)\Lambda}{\sqrt{mT^3(x)}}$ -- frequency of collisions; Λ -- Coulomb logarithm (it is assumed that $\nu \ll \omega$). Since $\Omega \sim \frac{c}{v_e}\omega_B$, the wavelength $\lambda = \frac{c}{\omega}$ is on the order of the layer thickness.

At the point where Re $\varepsilon = 0$ (region of plasma resonance), the wave electric field increases sharply ($E_y \sim E_0 \ln \varepsilon \sim E_0 \ln \frac{\omega}{\nu}$; $E_x \sim \frac{E_0}{\varepsilon} \sim \frac{\omega}{\nu}E_0$, /189
E_0 -- amplitude of incident wave). This leads to the fact that a considerable portion of the incident wave energy is absorbed in the layer having the thickness $\Delta x \sim \lambda\frac{\nu}{\omega}$, in the vicinity of the resonance point. Therefore, the intensity of braking radiation at the frequencies $\omega < \Omega$ is

$$I \sim I_{RJ}\,\Omega_0 \sim \frac{e^3 n_0^{3/2} T}{c^2 m^{3/2}}. \tag{8}$$

Consequently, the intensity of cyclotron radiation is $\frac{mc^2}{T}$ times less than the braking radiation intensity.

We may employ equation (7) only in the case of $\nu > \nu_0 \equiv \beta^{2/3}\,\Omega$. If $\nu < \nu_0$, we must take into account the formation of plasma waves in the resonance region.

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RADIATION OF LOW FREQUENCY WAVES BY IONS AND ELECTRONS OF A NON-ISOTHERMIC MAGNETOACTIVE PLASMA

V. I. Pakhomov

As is known, the propagation of three normal waves -- Alfvén, rapid and slow magnetosound waves -- is possible in a strongly non-isothermic ($T_e << T_i$) magnetoactive plasma, in the low frequency region ($\omega \lesssim \omega_{Hi}$). Each of these waves may be excited due to ion cyclotron radiation or due to Cherenkov radiation of ions and electrons moving along a spiral in this plasma.

This article determines the expressions for the intensities of radiation of these wave types by ions and electrons. The emissive and absorbant capacity of the plasma are determined in the frequency region $\omega \lesssim \omega_{Hi}$. The case of a low-pressure plasma is investigated in detail, /190
when the Alfvén velocity $v_A = \frac{H}{\sqrt{4\pi n_0 M}}$ considerably exceeds the speed of sound in the plasma $v_s = \sqrt{\frac{T_e}{M}}$. In this case, the refractive index of a slow magnetosound wave is considerably greater than the refractive indices of the two other waves. Therefore, it is natural to expect a sharp increase in the radiation intensity of a slow magnetosound wave. It is shown that in a low-pressure plasma the intensity of cyclotron radiation of a slow magnetosound wave by an ion for the s-th harmonic is $\left(\frac{v_A}{v_s}\right)^{2s+1}$ times greater than the radiation intensity of Alfvén waves and rapid magnetosound waves.

Cherenkov radiation of low frequency waves by electrons of a non-isothermic plasma may make a basic contribution to the over-all plasma radiation. It is shown that the ratio of the intensities of ion cyclotron radiation and Cherenkov electron radiation in the case of $\omega \approx s\omega_{Hi}$ is on the order of $\left(\frac{v_{Ti}}{v_\Phi}\right)^{2s-1}\left(\frac{v_{Te}}{v_\Phi}\right)$, where v_{Ti} and v_{Te} are the mean thermal velocities, respectively, of ions and electrons, and v_Φ is the phase velocity.*

Propagation of Electromagnetic Waves in a Non-Isothermic, Magnetoactive Plasma

The general expressions which are given in the appendix in (Ref. 2) may be employed to derive the dielectric constant tensor of a non-isothermic plasma located in the outer magnetic field. The sum with respect to particle types -- i.e., electrons and ions -- must be taken. Let us assume that the following condition is fulfilled

$$\mu_\alpha = \frac{k_\perp^2 v_{T\alpha}^2}{\omega_{H\alpha}^2} \ll 1, \quad \alpha = i,\ e;$$

$$z_{li} = \frac{\omega - l\omega_{Hi}}{\sqrt{2}\, k_\parallel v_{Ti}} \gg 1, \quad l = 0,\ \pm 1;$$

$$z_{le} = \frac{\omega - l\omega_{He}}{\sqrt{2}\, k_\parallel v_{Te}} \gg 1, \quad l = \pm 1,\ \pm 2, \ldots;$$

$$z_{0e} = \frac{\omega}{\sqrt{2}\, k_\parallel v_{Te}} \ll 1.$$

These conditions are fulfilled if it is assumed that the plasma is greatly non-isothermic ($T_e \gg T_i$). As a result, for the tensor of the plasma dielectric constant we obtain the following expression /191

$$\varepsilon_{ij}(\boldsymbol{k}, \omega) = a\delta_{ij} + ch_i h_j + d\varepsilon_{ijk} h_k + e(\varkappa_i [\varkappa \boldsymbol{h}]_j - \varkappa_j [\varkappa \boldsymbol{h}]_i) + f [\varkappa \boldsymbol{h}]_i [\varkappa \boldsymbol{h}]_j,$$

where

$$\boldsymbol{h} = \frac{\boldsymbol{H}}{H}; \quad \varkappa = \frac{\boldsymbol{k}}{k};$$

$$a = \frac{v}{u-1}; \quad c = \frac{v}{1-u} + v\left[\frac{n_s^2}{n^2 \cos^2\theta}\, q(z_{0e}) - 1\right];$$

$$d = -i\frac{v}{\sqrt{u}}\left[\frac{1}{1-u} + q(z_{0e})\,\mathrm{tg}^2\,\theta\right]\mathrm{sgn}\,\omega; \tag{1}$$

$$e = -i\frac{vq(z_{0e})}{\sqrt{u}\cos^2\theta}\,\mathrm{sgn}\,\omega; \quad f = i\sqrt{2\pi}\,\frac{mvn\beta_e}{Mu\cos\theta},$$

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* A portion of the results have been published in (Ref. 1).

where

$$v = \frac{\Omega_i^2}{\omega^2}; \quad u = \frac{\omega_{Hi}^2}{\omega^2}; \quad n_s = \frac{c}{v_s}; \quad n = \frac{kc}{\omega};$$

$$q(z_{0e}) = 1 + i\sqrt{\pi} z_{0e}; \quad \beta_e = \frac{v_{Te}}{c} = \sqrt{\frac{T_e}{mc^2}};$$

θ is the angle between the direction of wave propagation **k** and the direction of the outer magnetic field **H**. If the frequency ω is close to the harmonics of the ion gyrofrequency $s\omega_{Hi}$ (s = 2, 3, ...), the following expressions must be added to (1)

$$a' = -c' = -id' = 2i\sigma_s,$$

where

$$\sigma_s = \sqrt{\frac{\pi}{2}} \cdot \frac{vs^2 (s\beta n \sin\theta)^{2s-2}}{2^s s!\, \beta n\, |\cos\theta|} e^{-z_s^2},$$

$$z_s = \frac{\omega - s\omega_{Hi}}{\sqrt{2}\, k v_{Ti} \cos\theta}; \quad \beta = \frac{v_{Ti}}{c} = \sqrt{\frac{T^i}{Mc^2}}. \tag{2}$$

The dispersion equation determining the longitudinal refractive index $n_{\parallel} = n\cos\theta$ as the function of the transverse refractive index $n_{\perp} = n\sin\theta$ and the frequency ω has the following form in the given case [see also (Ref. 3)] /192

$$n_\parallel^6 + n_\parallel^4 \left(\frac{u-2}{u-1} n_\perp^2 - n_s^2 - \frac{2v}{u-1}\right) - n_\parallel^2 \left[n_\perp^2 \frac{n_\perp^2 + 2v}{u-1} + n_\perp^2 n_s^2 - \right.$$
$$\left. - \frac{v}{u-1}\left(n_A^2 + 2n_s^2\right)\right] + \frac{v}{u-1}\left[n_\perp^2\left(n_s^2 + n_A^2\right) - n_s^2 n_A^2\right] = iA, \tag{3}$$

where

$$A = \sqrt{\frac{\pi}{2}} \frac{1}{\beta_e n_\parallel} \left\{ n_s^2 \left[n_\parallel^4 + n_\parallel^2 \left(n_\perp^2 - \frac{2v}{u-1}\right) - \frac{v}{u-1}\left(n_\perp^2 - n_A^2\right) \right] + \right.$$
$$+ \frac{2n_\parallel^2 n_\perp^2 n_A^2}{u-1} + \frac{2n_\parallel^2 n_\perp^2 n_A^2}{n_s^2}\left(n_\parallel^2 - \frac{n_\perp^2 + v}{u-1}\right)\Bigg\} + 2i\sigma \left\{ n_\parallel^4 \left(\frac{n_\perp^2}{v} + 2\right) + \right.$$
$$+ n_\parallel^2 \left[\frac{(1+\sqrt{u})^2 + 2}{\sqrt{u}(1+\sqrt{u})} n_\perp^2 - 2n_s^2 + \frac{n_\perp^4}{v}\right] - n_\perp^2\left(n_s^2 + n_A^2\right) -$$
$$\left. - 2\frac{n_A^2 n_s^2 \sqrt{u}}{1+\sqrt{u}} \right\} \frac{n_\parallel^2}{v}; \quad \sigma = \sum_{s=-\infty}^{\infty} \sigma_s, \quad n_A = \frac{c}{v_A} = \frac{v}{u}.$$

When there is no damping (A = 0), equation (3) has three solutions $n_{\|} = n_{\|j}$ (j = 1, 2, 3). Assuming that

$$n_{\|} = n_{\|j} + in'_{\|j}, \quad n'_{\|j} \ll n_{\|j},$$

we obtain the following expression for $n'_{\|j}$

$$n'_{\|j} = \frac{A(n_{\|j})}{2n_{\|j}\left(n^2_{\|j} - n^2_{\|i}\right)\left(n^2_{\|j} - n^2_{\|k}\right)}, \quad j \neq i \neq k \neq j. \tag{4}$$

The damping coefficient of the j-th normal wave is

$$\varkappa_j = \frac{\omega_j}{c} n'_{\|j} \cos\chi, \tag{5}$$

where χ is the angle between the directions of the outer magnetic field and the energy flux of the corresponding wave.

The dispersion equation (3) may be solved in two limiting cases: If the Alfvén velocity is considerably greater than the speed of sound in the plasma ($v_A \gg v_s$) and if the frequency ω is considerably less than the ion gyrofrequency ($\omega \ll \omega_i$). /193

Let us examine the case when $v_A \gg v_s$. The dispersion equation (3) assumes the following form

$$\begin{gathered} n^6_{\|} + n^4_{\|}\left(\frac{u-2}{u-1}n^2_{\perp} - n^2_s\right) - n^2_{\|}\left[n^2_{\perp}\left(\frac{n^2_{\perp}}{u-1} + n^2_s\right) - \right. \\ \left. - 2n^2_s \frac{v}{u-1}\right] + n^2_s \frac{v}{u-1}\left(n^2_{\perp} - n^2_A\right) = 0. \end{gathered} \tag{6}$$

We may find one of the solutions for equation (6) by assuming that $n_{\|} \sim n_{\perp} \sim n_s \gg n_A$. As a result, we obtain (Ref. 4)

$$n^2_{\|1} = n^2_s + \frac{n^2_{\perp}}{u-1}. \tag{7}$$

The imaginary part of the refractive index of a slow magnetosound wave (7) is

$$n'_{\|1} = \frac{n^2_{\perp}}{n^4_{\|1}}\left(\sqrt{\frac{\pi}{8}} \cdot \frac{n^2_s}{\beta_e} + \sigma\frac{n_{\|1}n^2_{\perp}}{v}\right). \tag{8}$$

The first component in the right part of equation (8) takes into account Cherenkov absorption of a slow magnetosound wave in an electron gas. The second component takes into account cyclotron absorption in an ion gas.

We may find two other solutions for equation (6), corresponding to

Alfvén waves and rapid magnetosound waves, assuming that $n_{\parallel} \sim n_{\perp} \sim$ $\sim n_A \ll n_s$. The radiation of waves corresponding to these two solutions was studied in (Ref. 5, 6).

In another limiting case, when the frequency ω is considerably less than the ion gyrofrequency ω_{Hi} (region of strictly magnetohydrodynamic waves), the dispersion equation (3) assumes the following form

$$(n_\parallel^2 - n_A^2)\left[n_\parallel^4 - n_\parallel^2\left(n_s^2 + n_A^2 - n_\perp^2\right) - n_A^2 n_\perp^2 + n_s^2\left(n_A^2 - n_\perp^2\right)\right] = iA', \tag{9}$$

where

$$A' = \sqrt{\frac{\pi}{2}} \cdot \frac{1}{\beta n_\parallel n_s^2}\Big\{\left(n_\parallel^2 - n_A^2\right)\left[n_s^4\left(n_\parallel^2 - n_A^2 + n_\perp^2\right) + 2n_\parallel^2 n_\perp^2 n_A^2\right] + \frac{n_A^2}{u}\left[2n_\parallel^2 n_\perp^2\left(n_s^2 - n_A^2 - n_\perp^2\right) - n_s^4\left(2n_\parallel^2 - n_\perp^2 + n_A^2\right)\right]\Big\}. \tag{10}$$

The solution of equation (9) /194

$$n_{\parallel 1}^2 = n_A^2 + 0\left(\frac{1}{u}\right) \tag{11}$$

corresponds to an Alfvén wave. The imaginary part of the refractive index of an Alfvén wave is

$$n'_{\parallel 1} = \sqrt{\frac{\pi m}{8M}} \cdot \frac{2n_s^6\left(n_A^2 - n_\perp^2\right) + n_s^4\left(n_\perp^4 + n_A^4\right) + 2n_s^2 n_A^2\left(n_\perp^4 - n_A^4\right) - 2n_A^4\left(n_A^2 + n_\perp^2\right)^2}{u n_\perp^2 n_s^5} \tag{12}$$

Two other solutions (Ref. 7) of the dispersion equation (9)

$$n_{\parallel 2,3}^2 = \frac{1}{2}\left[n_A^2 + n_s^2 - n_\perp^2 \pm \sqrt{\left(n_s^2 - n_A^2 + n_\perp^2\right)^2 + 4n_A^2 n_\perp^2}\right] \tag{13}$$

determine the refractive indices of rapid waves and slow magnetosound waves. The imaginary parts of the refractive indices (13) determining the damping of magnetosound waves (Ref. 7) are

$$n'_{\parallel 2,3} = \sqrt{\frac{\pi m}{8M}} \cdot \frac{n_s^4\left(n_{\parallel 2,3}^2 - n_A^2 + n_\perp^2\right) + 2n_{\parallel 2,3}^2 n_A^2 n_\perp^2}{n_s n_{\parallel 2,3}^2\left(n_{\parallel 2,3}^2 - n_{\parallel 3,2}^2\right)}. \tag{14}$$

General Expression for Ion Radiation Intensity in a Non-Isothermic Plasma

Let us employ the general expressions (Ref. 4, 6) for determining the strengths of the electric and magnetic fields produced by an ion moving along a spiral in a strongly non-isothermic magnetoactive plasma. As a result, the components of the electromagnetic field produced by the ion in the wave guide zone assume the following form

$$
\begin{aligned}
E_{\varphi s}^{(j)} &= P_j \sin \Psi_{sj} \tilde{E}_{\varphi s}^{(j)}; \\
E_{xs}^{(j)} &= P_j \cos \Psi_{sj} \tilde{E}_{xs}^{(j)}; \\
H_{\varphi s}^{(j)} &= \pm P_j n_j \cos \Psi_{sj} \tilde{H}_{\varphi s}^{(j)}; \\
H_{xs}^{(j)} &= - n_j \cos(\chi \mp \theta) E_{\varphi s}^{(j)},
\end{aligned}
\tag{15}
$$

where /195

$$
P_j = \frac{2e\omega_{sj}}{c^2 R} \sqrt{\frac{n_\perp}{\sin\chi \left| \cos\chi \frac{d^2 n_{\parallel j}}{dn_\perp^2} \right|}} \; \frac{n_{\parallel j} e^{-\varkappa_{sj} R}}{v\left(n_{\parallel j}^2 - n_{\parallel l}^2\right)\left(n_{\parallel j}^2 - n_{\parallel k}^2\right)};
$$

$$
\Psi_{sj} = (\pm k_\perp \sin\chi + k_{\parallel j} \cos\chi) R - \omega_{sj} t \mp s\frac{\pi}{2} \mp \frac{\pi}{4} - s\varphi + \frac{\pi}{4} \operatorname{sgn}\left(\cos\chi \frac{d^2 n_{\parallel j}}{dn_\perp^2}\right);
$$

$$
\begin{aligned}
\tilde{E}_{\varphi s}^{(j)} &= v_\perp \left(n_\perp^2 \varepsilon_{11} + n_{\parallel j} \varepsilon_{33} - \varepsilon_{11}\varepsilon_{33}\right) J_s' - \Big[\frac{s v_\perp}{k_\perp r_0}\left(n_\perp^2 - n_{\parallel j} n_\perp - \right. \\
&\quad \left. - \varepsilon_{33}\right) i\varepsilon_{12} - v_\parallel \varepsilon_{11} i\varepsilon_{23} + v_\parallel n_{\parallel j} n_\perp (i\varepsilon_{12} - i\varepsilon_{23} \operatorname{ctg}\theta)\Big] J_s; \\
\tilde{E}_{xs}^{(j)} &= \pm \cos\chi \Big\{ v_\perp \left[n_\perp^2 (i\varepsilon_{12} - i\varepsilon_{23}\operatorname{ctg}\theta) - i\varepsilon_{12}\varepsilon_{33}\right] J_s' + \\
&\quad + \Big[\frac{s v_\perp}{k_\perp r_0}\left(n_j^2 n_\perp^2 - n_j^2 \varepsilon_{33} - n_\perp^2 \varepsilon_{11} + \varepsilon_{11}\varepsilon_{33} + \varepsilon_{23}^2\right) + v_\parallel n_{\parallel j} n_\perp \times \\
&\quad \times \left(n_j^2 - \varepsilon_{11}\right) + v_\parallel \varepsilon_{12}\varepsilon_{23}\Big] J_s \Big\} - \sin\chi \Big\{ v_\perp \left[n_{\parallel j} n_\perp (i\varepsilon_{12} - \right. \\
&\quad \left. - i\varepsilon_{23}\operatorname{ctg}\theta) + \varepsilon_{11} i\varepsilon_{23}\right] J_s' + \Big[\frac{s v_\perp}{k_\perp r_0}\left(n_j^2 n_{\parallel j} n_\perp - n_{\parallel j} n_\perp \varepsilon_{11} + \varepsilon_{12}\varepsilon_{23}\right) - \\
&\quad - v_\parallel \left(n_j^2 n_{\parallel j}^2 - n_j^2 \varepsilon_{11} - n_{\parallel j}^2 \varepsilon_{11} + \varepsilon_{12}\varepsilon_{23}\operatorname{ctg}\theta\right)\Big] J_s \Big\}; \\
\tilde{H}_{\varphi s}^{(j)} &= - v_\perp \cos\theta \left(i\varepsilon_{12}\varepsilon_{33} + \varepsilon_{11} i\varepsilon_{23} \operatorname{tg}\theta\right) J_s' - \\
&\quad - \Big[\frac{s v_\perp}{k_\perp r_0}\left(n_j^2 \varepsilon_{33} - \varepsilon_{11}\varepsilon_{33} - \varepsilon_{23}^2 + \varepsilon_{11}\varepsilon_{23}\operatorname{tg}\theta\right)\cos\theta + v_\parallel \varepsilon_{11} n_j^2 \sin\theta\Big] J_s.
\end{aligned}
\tag{16}
$$

In expressions (16), $v_\perp$ and $v_\parallel$ are, respectively, the ion velocity components which are perpendicular and parallel to the direction of the outer magnetic field; $r_0 = \frac{v_\perp}{\omega_{Hi}}$ -- Larmor radius of an ion; the argument of the Bessel function J_s and their derivatives J'_s equals $k_\perp r_0$. The radiated frequencies are

$$\omega_{sj} = s\omega_{Hi} + k_\parallel(\omega_{sj}) v_\parallel. \tag{17}$$

In the case of s = 0 (Cherenkov radiation), equation (17) assumes the following form

$$\beta_\parallel n_{\parallel j}(\omega_j) = 1. \tag{18}$$

In expressions (16) $n_\perp$ and $k_\perp$ are the positive solutions of the equation for the saddle point /196

$$\frac{dn_{\parallel j}(n_\perp)}{dn_\perp} \equiv \frac{dk_{\parallel j}(k_\perp)}{dk_\perp} = \mp \operatorname{tg}\chi. \tag{19}$$

The negative solutions correspond to saddle points which do not lie on the integration path, and therefore cannot make a contribution to the integral. The upper sign in equation (19), as well as in (16), corresponds to a cylindrically diverging wave; the lower sign corresponds to a cylindrically converging wave. When there are several positive solutions for equation (19), it is necessary to take the sum of all the solutions. Each solution for equation (19) determines the relationship between the angles χ and θ, i.e., the wave toward which the phase velocity is directed at the angle θ, and the group velocity at the angle χ to the direction of the outer magnetic field.

We may find the intensity of cyclotron ($s \neq 0$) and Cherenkov radiation of an ion per unit of solid angle w_{sj} by employing the following expression

$$w_{sj} = \frac{cR^2}{4\pi} \overline{\left(E^{(j)}_{\chi s} H^{(j)}_{\varphi s} - E^{(j)}_{\varphi s} H^{(j)}_{\chi s}\right)},$$

where the bar designates averaging over time. As a result, we obtain

$$w_{sj} = \frac{e^2\omega_{sj}^2}{2\pi c^3} \cdot \frac{n_j n_\perp n_{\parallel j}^2 U_{sj} e^{-2\varkappa_{sj} R}}{\sin\chi \left| \cos\chi \frac{d^2 n_{\parallel j}}{dn_\perp^2} \left(1 - \beta_\parallel \frac{d\omega n_{\parallel j}}{d\omega}\right) \right| v^2 \left(n_{\parallel j}^2 - n_{\parallel i}^2\right)^2 \left(n_{\parallel j}^2 - n_{\parallel k}^2\right)^2}, \tag{20}$$

where

$$U_{sj} = \tilde{E}^{(j)}_{\chi s} \tilde{H}^{(j)}_{\varphi s} + \cos(\chi \mp \theta) \left(\tilde{E}^{(j)}_{\varphi s}\right)^2.$$

In the case of $s = 2, 3, \ldots$, formula (20) is valid for the cases of slow ($v_\parallel \sim v_{Ti}$) and rapid ($v_\parallel \gg v_{Ti}$) ions. In order to determine the radiation intensity at the main harmonics ($s = 1$), expression (20) may be only employed in the case of rapid ions. In the case of $s = 1$, the results given in (Ref. 4) must be employed to determine the radiation intensity of slow ions. For Cherenkov radiation ($s = 0$), formula (20) is also valid only in the case of rapid ions. Cherenkov radiation of slow ions is greatly absorbed.

Expression (20) for ions with a velocity on the order of the mean thermal velocity of plasma ions has the following form (in the case of $s = 2, 3, \ldots$) /197

$$w_{sj} = \frac{e^2\omega_{sj}^2\beta_\perp^2}{2\pi c} U_{sj} e^{-2\varkappa_{sj}R}, \tag{21}$$

where

$$U_{sj} = \frac{s^2 (sn_\perp\beta_\perp)^{2s-2} n_j n_\perp n_{\parallel j}^2}{2^{2s}(s!)^2 \sin\varkappa \left|\cos\varkappa \frac{d^2 n_{\parallel j}}{dn_\perp^2}\right| v^2 (n_{\parallel j}^2 - n_{\parallel i}^2)^2 (n_{\parallel j}^2 - n_{\parallel k}^2)^2} \times$$

$$\times \{[\mp n_j^2 \sin(\chi \mp \theta)(n_j n_\perp - \varepsilon_{11}\sin\theta + i\varepsilon_{12}\sin\theta - i\varepsilon_{23}\cos\theta) + (\varepsilon_{11} -$$

$$- i\varepsilon_{12})(\varepsilon_{33}\cos\chi \mp i\varepsilon_{23}\sin\chi) + \varepsilon_{23}^2\cos\chi - \varepsilon_{33}n_j^2\cos\chi]\cos\theta[(\varepsilon_{11} -$$

$$- i\varepsilon_{12})(\varepsilon_{33} - i\varepsilon_{23}\operatorname{ctg}\theta) - n_j^2\varepsilon_{33} + \varepsilon_{23}^2] + \cos(\chi \mp \theta)[(\varepsilon_{11} - i\varepsilon_{12}) \times$$

$$\times (\varepsilon_{33} - n_\perp^2) - n_j n_{\parallel j}(\varepsilon_{33}\cos\theta + i\varepsilon_{23}\sin\theta)]^2\}.$$

Averaging expression (21) over the ion distribution, which is assumed to be a Maxwell distribution, we may determine the contribution made by ion cyclotron radiation to the emissive power of the plasma at the $\omega \approx s\omega_{Hi}$ frequencies

$$\eta_{sj} = \frac{2^s s! e^2 \omega n_0}{(2\pi)^{3/2} c n_{\parallel j}} U_{sj} \frac{\beta^{2s-1}}{\beta_\perp^{2s-2}} e^{-z_s^2}, \tag{22}$$

where n_0 is the plasma density.

Cyclotron Radiation of a Slow Magnetosound Wave in a Low-Pressure Plasma

It was noted above that the dispersion equation (3) may be solved if the magnetic pressure p_H is considerably greater than the gasokinetic pressure of electrons $p_e = n_0 T_e$ (this is equivalent to the condition $v_A \gg v_S$). Let us investigate the cyclotron radiation of a slow magnetosound wave corresponding to the solution (7) of the dispersion equation.

Employing the general expression (20) for the ion radiation intensity and taking the condition $n_A \ll n_s$ into account for the intensity of the radiation of a slow magnetosound wave by an ion at the s-th harmonics per unit of solid angle, we obtain the following expression

$$w_{s1} = \frac{e^2\omega_{s1}^2}{2\pi c^3} U_{s1} e^{-2\varkappa_{s1} R}, \tag{23}$$

where /198

$$U_{s1} = \frac{n_1^2 \sin\theta}{v^2 n_{\parallel 1}^6 \sin\chi \left| \cos\chi \frac{d^2 n_{\parallel 1}}{dn_\perp^2}\left(1 - \beta_\parallel \frac{d\omega n_{\parallel 1}}{d\omega}\right)\right|} \{n_1^2 \sin(\chi - \theta)[v_\perp \sin\theta \times$$

$$\times (i\varepsilon_{12} - i\varepsilon_{23}\operatorname{ctg}\theta) J_s' + \left(\frac{sv_\perp}{k_\perp r_0}\sin\theta - v_\parallel\cos\theta\right) n_1^2 J_s\Big]\Big[v_\perp(\varepsilon_{33} i\varepsilon_{12} +$$

$$+ \varepsilon_{11} i\varepsilon_{23}\operatorname{tg}\theta) J_s' + \left(\frac{sv_\perp}{k_\perp r_0}\varepsilon_{33}\cos\theta - v_\parallel\varepsilon_{11}\sin\theta\right) n_1^2 J_s\Big] + \cos(\chi - \theta) \times$$

$$\times \Big[v_\perp(\varepsilon_{11} n_\perp^2 + \varepsilon_{33} n_{\parallel 1}^2) J_s' - (i\varepsilon_{12} - i\varepsilon_{23}\operatorname{ctg}\theta)\left(\frac{sv_\perp}{k_\perp r_0}\operatorname{tg}\theta + v_\parallel\right) \times$$

$$\times n_{\parallel 1} n_\perp J_s\Big]^2\Big\}.$$

In the case under consideration, the refractive index is determined by formula (7), and the damping coefficient is

$$\varkappa_{s1} = \frac{\omega_{s1} n_1^2 \cos\chi}{c n_{\parallel 1}^4}\left(\sqrt{\frac{\pi}{8}} \cdot \frac{n_s^2}{\beta_e} + \frac{\sigma_s}{v} n_{\parallel 1} n_\perp^2\right). \tag{24}$$

For ions whose velocity is on the order of the mean thermal velocity of plasma ions, expression (23) may be simplified (in the case of s = 2, 3, ...):

$$w_{s1} = \frac{e^2\omega_{s1}^2\beta_\perp^2}{2\pi c} U_{s1} e^{-2\varkappa_{s_1} R}, \tag{25}$$

where

$$U_{s1} = \frac{s^2 (s\beta_\perp n_\perp)^{2s-2} n_\perp^4 n_1^5 \sin(\chi - \theta)}{2^{2s}(s!)^2 v (u - 1) n_{\parallel 1}^7 \sin\chi \left|\cos\chi \frac{d^2 n_{\parallel 1}}{dn_\perp^2}\right|}.$$

In this case, the equation for the saddle point (19) yields the following dependence between the angles χ and θ:

$$\operatorname{tg}\theta = (u - 1)\operatorname{tg}\chi. \tag{26}$$

The contribution resulting from magneto-braking radiation of a slow magnetosound wave by ions to the emissive power of the plasma, for frequencies close to $s\omega_{Hi}$, is

$$\eta_{s1} = \frac{e^2 \omega_{s1} n_0 s (s\beta n_{\perp})^{2s-1} n_{\perp}^3 n_1^5 \sin(\theta - \chi)}{(2\pi)^{3/2} 2^s s!\, v c n_s^2 n_{\parallel 1}^5 \sin\chi\, |\cos\chi|} e^{-z_s^2}. \tag{27}$$

In order of magnitude, $w_{s_1} \sim w_0 n_A \left(\frac{v_{Ti}}{v_s}\right)^{2s-2} \left(\frac{v_A}{v_s}\right)^3$; where w_0 is the total intensity of ion radiation in a vacuum. It was shown in (Ref. 6) that the intensities of cyclotron radiation of an Alfvén wave and of a rapid magnetosound wave ($j = 2, 3$) are /199

$$w_{s2,3} \sim w_0 n_A \left(\frac{v_{Ti}}{v_A}\right)^{2s-2}.$$

Comparing these results, we find that

$$\frac{w_{s1}}{w_{s2,3}} \sim \left(\frac{v_A}{v_s}\right)^{2s+1} \gg 1.$$

Thus, in a greatly non-isothermic plasma having a low density, the intensity of cyclotron radiation of a slow magnetosound wave for the s-th harmonics is, in order of magnitude, $\left(\frac{v_A}{v_s}\right)^{2s+1}$ times greater than the radiation intensity of an Alfvén wave and a rapid magnetosound wave.

Cherenkov Radiation of Magnetohydrodynamic Waves by Ions

In the case of $\omega \ll \omega_{Hi}$, the dispersion equation (3) has three solutions corresponding to magnetohydrodynamic waves. The refractive indices of these waves are on the order of n_s (or n_A), and are determined by expressions (11), (12). It follows from the condition of Cherenkov radiation $\beta_{\parallel} n_{\parallel j} = 1$ that frequencies $\omega \ll \omega_{Hi}$ may exist during radiation of rapid ions, whose velocity is

$$\beta_{\parallel} \sim \beta \sqrt{\frac{T_e}{T_i}}.$$

Let us investigate Cherenkov radiation of ions in the low-frequency region $\omega \ll \omega_{Hi}$ ($u \gg 1$). In order to determine the Cherenkov radiation intensity, let us employ the general expressions (20), assuming that $s = 0$ and $u \gg 1$. In addition, let us assume that the arguments of the Bessel function are small

$$k_{\perp} r_0 = \frac{n_{\perp} \beta_{\perp}}{\sqrt{u}} \ll 1. \tag{28}$$

Taking into account (28) for the Cherenkov radiation intensity of an Alfvén wave, we obtain the following /200

$$w_{01} = \frac{e^2 \omega_{01}}{8\pi v_{\parallel}} \cdot \frac{n_1^2 \sin^3 \theta \left[n_A^2 \beta_{\perp}^2 (n_1^2 - n_s^2) + 2n_1^2 \right]^2}{u^2 \cos\theta \sin\chi \left| \frac{d^2 n_{\parallel 1}}{dn_{\perp}^2} \cdot \frac{dn_{\parallel 1}}{d\omega} \right| (n_A^2 - n_{\parallel 2}^2)^2 (n_A^2 - n_{\parallel 3}^2)^2} e^{-2\varkappa_{01} R}. \tag{29}$$

The damping coefficient of an Alfvén wave is

$$\varkappa_{01} = \frac{\omega_{01}}{c} \cos\chi \sqrt{\frac{\pi m}{8M}} \cdot \frac{2n_s^6 (n_A^2 - n_{\perp}^2) + n_s^4 (n_A^4 + n_{\perp}^4) + 2n_s^2 n_A^2 (n_{\perp}^4 - n_A^4) + 2n_A^4 (n_A^2 + n_{\perp}^2)^2}{u n_{\perp}^2 n_s^5} \tag{30}$$

Equation (19) for the saddle point assumes the following form in this case

$$n_{\perp}^4 - \frac{u n_s^2}{n_A} n_{\perp}^3 \operatorname{tg}\chi + n_A^2 (n_s^2 - n_A^2) = 0. \tag{31}$$

It follows from equation (30) that the Alfvén wave is radiated within the limits of a narrow cone $\left(\chi \sim \frac{1}{u} \right)$ along the direction of the outer magnetic field.

The Cherenkov radiation intensity of rapid and slow magnetosound waves per unit of solid angle, when condition (28) is fulfilled, is

$$w_{02,3} = \frac{e^2 \omega_{02,3}}{8\pi v_{\parallel}} \cdot \frac{n_{\parallel 2,3}^2 \sin^3\theta \cos(\chi \mp \theta) \left[\beta_{\perp}^2 (n_{\parallel 2,3}^2 - n_s^2) + 2 \right]^2}{u \cos^2\theta \sin\chi \left| \cos\chi \frac{d^2 n_{\parallel 2,3}}{dn_{\perp}^2} \cdot \frac{dn_{\parallel 2,3}}{d\omega} \right| (n_{\parallel 2}^2 - n_{\parallel 3}^2)^2} e^{-2\varkappa_{02,3} R}. \tag{32}$$

The damping coefficients of magnetosound waves are determined by the expression

$$\varkappa_{02,3} = \frac{\omega_{02,3}}{c} \cos\chi \sqrt{\frac{\pi m}{8M}} \cdot \frac{n_s^4 (n_{\parallel 2,3}^2 - n_A^2 + n_{\perp}^2) + 2n_{\parallel 2,3}^2 n_A^2 n_{\perp}^2}{n_s n_{\parallel 2,3}^2 (n_{\parallel 2,3}^2 - n_{\parallel 3,2}^2)}. \tag{33}$$

The figure presents graphs of the functions $\frac{dn_{\parallel j}}{dn_{\perp}}$ and $n^2_{\parallel j}(n^2_{\perp})$ for the case $n_{\parallel} > 0$ and $n_A > n_s$. The points at which the curve $\frac{dn_{\parallel j}}{dn_{\perp}}$ intersects the line $y = \mp \mathrm{tg}\chi$ correspond to the solution of equation (19). /201
As may be seen from the figure, a slow magnetosound wave (j = 2) represents a cylindrically converging wave. There are thus two waves for each angle $\chi < \chi_{max}$. In the case of $\chi > \chi_{max}$, a slow magnetosound wave is not radiated. The angle $\chi = \chi_{max}$ is determined by the following equations

$$\frac{dn_{\parallel 2}}{dn_{\perp}} = \mathrm{tg}\,\chi_{max} \quad \frac{d^2 n_{\parallel 2}}{dn^2_{\perp}} = 0.$$

The rapid magnetosound wave (j = 3) represents a cylindrically diverging wave. For any angle χ, one such wave is radiated. Expression (32) for a slow magnetosound wave is not valid in the limiting case, when $n_{\perp} \to \infty$ and $n^2_{\parallel} \to n^2_A + n^2_s$. In this case, we must employ the general expression (20) for $u \gg 1$ and $s = 0$. (We should point out that Cherenkov radiation in a direction which is perpendicular to the direction of the outer magnetic field may be absent since the condition $\beta_{\parallel} n_{\parallel} = 1$ thus contradicts the assumption $\omega \ll \omega_{Hi}$.) If $n_A < n_s$, then the substitution $n_A \rightleftarrows n_s$ must be made in the figure. In the case of $n_{\parallel} < 0$, the functions $\frac{dn_{\parallel j}}{dn_{\perp}}$ change their sign.

Electron Cherenkov Radiation

It is of interest to study Cherenkov radiation of low frequency waves ($\omega \sim \omega_{Hi}$) by electrons of a non-isothermic plasma, because under certain conditions it may make the main contribution to the over-all plasma radiation. In particular, this is related to the fact that the intensity of ion cyclotron radiation sharply decreases as one recedes from the center of the line $\omega = s\omega_{Hi}$ (s = 1, 2, ...).

We obtained the expressions for the intensity of Cherenkov radiation, by an electron moving in the plasma along a spiral, of each of three normal waves. We determined the contribution made by Cherenkov electron radiation to the emissive power of a non-isothermic plasma at the $\omega \sim \omega_{Hi}$ frequencies. /202

We shall employ the general expressions given in (Ref. 2) for s = 0 in order to determine the components of an electromagnetic field

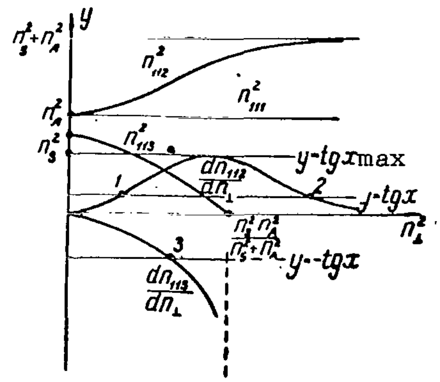

Figure 1

produced by an electron in the wave zone. As a result, we obtain

$$
\begin{aligned}
E_{\varphi j} &= P_j \sin\left(\Psi + \alpha\frac{\pi}{4}\right)\tilde{E}_{\varphi i};\\
E_{\chi j} &= P_j \cos\left(\Psi + \alpha\frac{\pi}{4}\right)\tilde{E}_{\chi i};\\
H_{\varphi j} &= \pm P_j n_j \cos\left(\Psi + \alpha\frac{\pi}{4}\right)\tilde{H}_{\varphi i};\\
H_{\chi j} &= -n_j \cos(\chi \mp \theta) E_{\varphi j},
\end{aligned}
\tag{34}
$$

where

$$\Psi = R\sin\chi\,[k_\perp \pm k_{\parallel j}(k_\perp)\operatorname{ctg}\chi] \mp \omega t - \frac{\pi}{4};$$

$$\alpha = \operatorname{sgn}\left(\cos\chi\,\frac{d^2 n_{\parallel j}}{dn_\perp^2}\right);$$

$$
\begin{aligned}
\tilde{E}_{\varphi i} &= v_\perp\,(n_\perp^2\varepsilon_{11} + n_{\parallel j}^2\varepsilon_{33} - \varepsilon_{11}\varepsilon_{33})\,J_0' - v_\parallel\,[n_{\parallel j}n_\perp\,(i\varepsilon_{12} - i\varepsilon_{23}\operatorname{ctg}\theta) + \\
&\quad + \varepsilon_{11}i\varepsilon_{23}]\,J_0;\\
\tilde{E}_{\chi i} &= \pm\cos\chi\,\{v_\perp[n_\perp^2\,(i\varepsilon_{12} - i\varepsilon_{23}\operatorname{ctg}\theta) + i\varepsilon_{12}\varepsilon_{33}]\,J_0' - v_\parallel[n_{\parallel j}n_\perp(n_j^2 - \\
&\quad - \varepsilon_{11}) + \varepsilon_{12}\varepsilon_{23}]\,J_0\} - \sin\chi\,\{v_\perp\,[n_{\parallel j}n_\perp\,(i\varepsilon_{12} - i\varepsilon_{23}\operatorname{ctg}\theta) + \varepsilon_{11}i\varepsilon_{23}]J_0' + \\
&\quad + v_\parallel[n_j^2 n_{\parallel j}^2 - n_j^2\varepsilon_{11}(1+\cos^2\theta) + \varepsilon_{11}^2 + \varepsilon_{12}^2]J_0\};\\
\tilde{H}_{\varphi i} &= -v_\perp\cos\theta\,(i\varepsilon_{12}\varepsilon_{33} + \varepsilon_{11}i\varepsilon_{23}\operatorname{tg}\theta)\,J_0' + v_\parallel\sin\theta\,[\varepsilon_{11}(n_j^2 - \\
&\quad - \varepsilon_{11}) + \varepsilon_{12}\varepsilon_{23}\operatorname{ctg}\theta - \varepsilon_{12}^2]\,J_0.
\end{aligned}
\tag{35}
$$

In (34), $v_\parallel$ and $v_\perp$ are, respectively, the electron velocity components which are parallel and perpendicular to the direction of the outer magnetic field. The argument of the Bessel functions and their derivatives equals $k_\perp r_0$, where $r_0 = \frac{v_\perp}{\omega_{He}}$ is the Larmor electron radius. The remaining notation was presented above.

Employing formulas (34) and (35), we may obtain the following expression for the intensity of electron Cherenkov radiation per unit of solid angle /203

$$w_j = \frac{n_j c R^2 P_j^2}{2\pi\omega\beta_\parallel \left|\frac{dn_{\parallel j}}{d\omega}\right|} \left[\tilde{E}_{\chi j}\tilde{H}_{\varphi j} + \cos(\chi \mp \theta)\left(\tilde{E}_{\varphi j}\right)^2\right]. \tag{36}$$

The argument of the Bessel function is small for electrons having a velocity $v_\perp$ which is on the order of the mean thermal velocity of plasma electrons ($v_\perp \sim v_{Te} = \sqrt{\frac{T_e}{m}}$):

$$k_\perp r_0 = \frac{m}{M}\cdot\frac{n_\perp\beta_\perp}{\sqrt{u}} \ll 1.$$

In this case, expression (36) may be simplified

$$w_j = \frac{e^2\omega_j}{2\pi v_\parallel}\cdot\frac{n_j n_\perp U_j e^{-2\varkappa_{0j}R}}{v(1-u)\sin\chi\left|\cos\chi\frac{d^2n_{\parallel j}}{dn_\perp^2}\cdot\frac{dn_{\parallel j}}{d\omega}\right|\left(n_{\parallel j}^2 - n_{\parallel i}^2\right)^2\left(n_{\parallel j}^2 - n_{\parallel k}^2\right)^2}, \tag{37}$$

where

$$U_j = n^2\sin\theta\left\{\cos\chi\left[\frac{v^2\,\mathrm{tg}^2\,\theta}{u(1-u)} - n_{\parallel j}n_\perp\left(n_j^2 + \frac{v}{1-u}\right)\right] \pm \sin\chi \times\right.$$
$$\left.\times\left[n_{\parallel j}^2 n_j^2 + \frac{vn_j^2}{1-u}(1+\cos^2\theta) - \frac{v^2}{u(1-u)}\right]\right\} + \frac{v\cos(\chi\mp\theta)}{u(1-u)}\times$$
$$\times\left(\frac{v}{1-u} - un_{\parallel j}n_\perp\right)^2.$$

Averaging (37) over the electron distribution, which is assumed to be a Maxwell distribution, we may find the contribution made by electron Cherenkov radiation to the plasma emissive power at the frequencies $\omega \sim \omega_{Hi}$:

$$\eta_j = \frac{e^2\omega n_0}{(2\pi)^{3/2}v_{Te}}\cdot\frac{n_j\,\mathrm{tg}\,\theta\cdot U_j}{v(1-u)\sin\chi\left|\cos\chi\frac{d^2n_{\parallel j}}{dn_\perp^2}\right|\left(n_{\parallel j}^2 - n_{\parallel i}^2\right)^2\left(n_{\parallel j}^2 - n_{\parallel k}^2\right)^2}. \tag{38}$$

Comparing the contribution made, in order of magnitude, by ion cyclotron radiation (22) and by electron Cherenkov radiation to the plasma emissive power, we obtain

$$\frac{\eta_{sj}}{\eta_j} \sim \left(\frac{v_{Ti}}{v_\Phi}\right)^{2s-1}\left(\frac{v_{Te}}{v_\Phi}\right). \tag{39}$$

It can be seen from (39) that the relative contribution of the electron Cherenkov radiation increases with an increase in the harmonic number s, since it was assumed from the beginning that $v_{Ti} \ll v_\Phi \ll v_{Te}$. /204

Let us investigate the Cherenkov radiation of a slow magnetosound wave by electrons in a low-pressure plasma ($n_s \gg n_A$). We shall assume that the argument of the Bessel functions in (35) is small ($k_\perp r_0 \ll 1$). This can be fulfilled only for very rapid electrons ($v_\perp \gg v_{Te}$). Employing the general expressions for the electromagnetic field components (34), (35), the equations (7), (26), and also the condition of electron Cherenkov radiation, we finally obtain

$$w_1 = \frac{e^2\omega_{Hi}^2}{2\pi v_\parallel}\cdot\frac{[\mathrm{tg}^2\chi + (1-\beta_\parallel^2 n_s^2)^2]^2(1-\beta_\parallel^2 n_s^2)^2}{\mathrm{tg}^4\chi\,(\sec^2\chi - \beta_\parallel^2 n_s^2)^2}\cdot\frac{e^{-2\chi_{01}R}}{\beta_\parallel^2 n_s^2 n_A^2\,|\cos\chi|}. \tag{40}$$

Expression (40) determines the intensity of Cherenkov radiation for an electron having the velocity $\beta_\parallel$ per unit of solid angle in the direction χ. If $\beta_\parallel n_s < 1$, then the radiated wave represents a cylindrically converging wave (for $\chi < \frac{\pi}{2}$). The waves may thus be radiated in any direction $\times$. The frequency of a wave radiated at the angle χ is

$$\omega = \frac{\omega_{Hi}\,\mathrm{tg}\,\chi}{\sqrt{|\sec^2\chi - \beta_\parallel^2 n_s^2\chi|}}. \tag{41}$$

In another limiting case, when $\beta_\parallel n_s > 1$, the radiated wave represents a cylindrically diverging wave. The vector of the group velocity is thus directed at the angle $\chi > \chi_{min}$, where

$$\chi_{min} = \mathrm{arc\,sec}\,(\beta_\parallel n_s).$$

The phase velocity in this case is directed at the angle $\theta < \theta_{max}$ in the direction of the outer magnetic field, while $\theta_{max} = \chi_{min}$. However, we cannot employ the expressions obtained in the case of $\chi \to \chi_{min}$, since in this case $\omega \to \infty$, according to (41).

Averaging (40) over the electron distribution, we obtain the contribution produced by Cherenkov radiation, by plasma electrons, of a slow magnetosound wave to the plasma emissive power:

$$\eta_1 = \frac{e^2\omega_{Hi} n_0 \left[\mathrm{tg}^2\chi + \left(1 - \beta_\parallel^2 n_s^2\right)^2\right]\left(1 - \beta_\perp^2 n_s^2\right)^2}{(2\pi)^{3/2} v_{Te} \beta_\parallel^2 n_s^2 n_A^2 \sin\chi \sqrt{\left|\sec^2\chi - \beta_\parallel^2 n_s^2\right|}} \tag{42}$$

We should recall that at any angle $\chi > \chi_{min}$, electromagnetic waves with two frequencies are radiated -- one with the frequency $\omega > \omega_{Hi}$ (electrons with $\beta_\parallel n_s > 1$) and the other with the frequency $\omega < \omega_{Hi}$ (electrons with the velocity $v_\parallel < v_s$). Expressions have been obtained (Ref. 6), in which it must be assumed that $z_{0e} << 1$, for the intensity of Cherenkov radiation by electrons of an Alfvén wave and a rapid magnetosound wave in a non-isothermic plasma with a low density. /205

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BRAKING OF RELATIVISTIC PARTICLES IN LOW ATMOSPHERIC LAYERS

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As is known, when a relativistic charged particle moves in an external field, the braking force by radiation increases proportionally to the square of the particle energy, and may be greater than the Lorentz force (Ref. 1). The influence of the medium on the braking radiation is of great interest, since -- when particles move in the lower atmospheric layers -- the braking force by radiation may differ significantly from the vacuum force. The expression for the braking force by radiation of an oscillator in a medium was obtained and studied in the non-relativistic approximation in (Ref. 2, 3). The spectral radiation density was studied in the relativistic case in (Ref. 4 - 7).

In order to investigate the influence of the medium on the braking radiation, it is necessary to know the total energy losses of the particle, i.e., it is necessary to specify the medium properties. We shall formulate a model for a medium with an isotropic dielectric with a given dispersion /206

$$\varepsilon(\omega) = 1 - \frac{\omega_0^2}{\omega^2 - \Omega^2},$$

where Ω are the atom resonance frequencies; $\omega_0^2 = \frac{4\pi e^2 n_0}{m}$; n_0 -- density of the medium particles.

The following expression was obtained in (Ref. 4 - 7) for the spectral density of particle radiation in the frequency region $\omega >> \omega_H^*$ ($\omega_H^* = \omega_H \sqrt{1 - \beta^2}$)

$$P(\omega) = -\frac{1}{\pi\sqrt{3}} \cdot \frac{e^2 v\omega}{c^2}\left(1 - \frac{1}{\beta^2\varepsilon}\right)\left\{2K_{\frac{2}{3}}(\xi) - \int_\xi^\infty K_{\frac{1}{3}}(\eta)\,d\eta\right\}, \beta^2\varepsilon < 1;$$

$$P(\omega) = \frac{e^2 v\omega}{c^2}\left(1 - \frac{1}{\beta^2\varepsilon}\right)\left\{1 + 2\left[J_{-2/3}(\xi) - J_{2/3}(\xi)\right] - \int_\xi^\infty \left[J_{-1/3}(\eta) - J_{1/3}(\eta)\right] d\eta\right\}, \ \beta^2\varepsilon > 1, \quad (1)$$

where $\xi = \frac{2}{3} \cdot \frac{\omega}{\omega_H^*}(1 - \beta^2\varepsilon)^{3/2}$; $J_p(\xi)$ and $K_p(\xi)$ are the Bessel functions.

Let us employ formula (1) to determine the total losses by radiation as a function of the particle energy. Since it is difficult to integrate equation (1) in the general case, let us find the frequency regions in which the spectral density has the maxima ($\xi \sim 1$), and let us determine the radiation intensity in each of these regions.

1. In the low-energy region, when the following inequality is fulfilled

$$\frac{\omega_H}{\Omega} \ll \frac{\omega_0^2}{\Omega^2} \ll \frac{1}{E^2} \ll 1, \; E = (1 - \beta^2)^{-1/2}, \tag{2}$$

the frequency $\omega_m \sim \omega_H (1 - \beta^2)^{-1}$ -- which makes the maximum contribution to the vacuum (Ref. 8, 9) -- is small as compared with the resonance frequency $\omega_m \ll \Omega$. In this case, the radiation spectral intensity has maxima at the frequencies ω_m and Ω. The braking radiation at the frequencies ω_m causes losses which coincide with vacuum losses

$$W_B \sim \frac{e^2 v}{c^2} \cdot \frac{\omega_H^2}{E^2}. \tag{3}$$

The maximum close to the frequency determined from condition $\beta^2 \varepsilon(\omega) \approx 1$ is also caused by the curvature of the particle trajectory. However, in this case the influence of the medium is significant, as a result of which the radiation is considerably greater than in the preceding case /207

$$W \sim \frac{e^2 v}{c^2} \omega_0^2 \left(\frac{\omega_H E^2}{\Omega}\right)^{2/3}. \tag{4}$$

Thus, the braking radiation is larger (in the energy region under consideration) than in a vacuum, and increases with the energy E proportionally to $E^{4/3}$.

2. The inequality (2) assumes the following form for given characteristics of the medium and unchanged magnetic field strength with an increase in the particle energy:

$$\frac{\omega_H}{\Omega} \ll \frac{1}{E^2} \ll \frac{\omega_0^2}{\Omega^2} \ll 1. \tag{5}$$

In this case, the braking radiation maximum is located at the frequency $\omega \simeq \frac{\Omega^3 \omega_H}{\omega_0^3} \cdot \frac{1}{E}$. The corresponding loss is

$$W \sim \frac{e^2 v}{c^2} \cdot \frac{\omega_H^2 \Omega^2}{\omega_0^4} \cdot \frac{1}{E^2}. \tag{6}$$

3. With an increase in the particle energy, the relationship between the particle parameters and the medium assumes the following form

$$\frac{1}{E^2} \ll \frac{\omega_H}{\Omega} \ll \frac{\omega_0^2}{\Omega^2} \ll 1. \qquad (7)$$

Thus, the radiation maximum in a vacuum occurs in the frequency region $\omega \gg \Omega$. The force of the braking radiation depends on the parameter $\mu = \frac{\omega_0}{\omega_H} \cdot \frac{1}{E}$ in this case.

In the case of $\mu \gg 1$, the braking radiation intensity in the frequency region $\omega \sim \omega_m$ is exponentially small as compared with the corresponding value in a vacuum. The braking radiation intensity is also small $\left(\frac{W}{W_в} \sim \frac{\Omega^4}{\omega_0^4 E^4}\right)$. In the opposite case ($\mu \ll 1$), the influence of the medium on the braking radiation may be disregarded, so that the particle losses coincide with vacuum losses.

Thus, the medium influences the particle braking radiation only when the inequality $\omega_0^2 \gg \Omega\omega_H$ is fulfilled. In the region of relatively small energies $\left(E^2 \ll \frac{\Omega^2}{\omega_0^2}\right)$, the presence of the medium leads to an increase /208 in the braking radiation, and in the energy region $E^2 \gg \frac{\Omega^2}{\omega_0^2}$ it leads to a decrease in the braking radiation, as compared with a vacuum. Thus, the braking radiation is significantly less than the vacuum radiation in the energy region $\frac{\omega_0}{\omega_H} \gg E \gg \left(\frac{\Omega}{\omega_H}\right)^{1/2}$.

The change in the braking radiation intensity in a medium may be explained in physical terms as follows. In the presence of the medium, the frequency determining the maximum of the radiation spectral density, according to formulas (1), depends on the medium parameters

$$\omega_m^*(\varepsilon) = \frac{\omega_H^*}{(1-\beta^2\varepsilon)^{3/2}}, \quad \omega_m^*(1) = \omega_m.$$

For small particle energies, the presence of the medium leads to an increase in the frequency ω_m(ε $(\omega_m) > 1$, and the corresponding wavelength $\lambda_\partial = \frac{\lambda_0}{\sqrt{\varepsilon}}$ decreases): $\omega_m < \omega_m^* < \Omega$. Therefore, the braking radiation intensity, which is proportional to $(\omega_m^*)^{4/3}$, also increases. For large particle energies ($\omega_m >> \Omega$), the frequency ω_m^* decreases considerably: $\omega_m^* << \Omega << \omega_m$ (the characteristic wavelength λ_m increases, $\varepsilon(\omega_m) < 1$). Correspondingly, the braking radiation intensity decreases. The Cherenkov radiation, which can be computed from the customary formulas in the case under consideration, is larger than the braking radiation throughout the entire energy region where the influence of the medium must be taken into account.

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EXCITATION OF WAVES IN A CONFINED PLASMA
BY MODULATED CURRENTS

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The excitation of waves in a plasma by modulated currents has been studied repeatedly (Ref. 1 - 3). In all of these studies, it was assumed that the plasma was an unconfined, isothermic, and linear medium. In this article we shall study the excitation of waves in a confined plasma, and we shall attempt to take its nonlinearity into account. /209

When solving the problem of wave excitation in a plasma by a modulated current j, we assume that the current is given, and we shall disregard the inverse influence of the wave on the motion of current particles. This is valid as long as the energy losses of the current particles at the wave length are negligibly small as compared with the energy itself. The electric field strength of the wave is determined by the equation $LE = j$, where L is the differential operator, which is generally speaking nonlinear. A solution of this equation leads to the following value for the Fourier field components: $E = \frac{j}{\Delta}$. If the frequency of the current modulation is such that Δ vanishes (dispersion equation), then the field strength becomes large. It is usually assumed that Δ is limited by particle collisions or Cherenkov absorption of wave energy by plasma particles. This is the result of the linear theory, and it is valid for insignificant amplitudes of the wave field strength. However, in resonance, when $\Delta \sim 0$, the field strength greatly increases, and the linear theory may be invalid. Even in the case of slight nonlinearity, the limitation on the amplitude of the wave field strength due to nonlinear interaction may be more substantial than the limitation imposed by dissipation.

Let us study the plasma layer which is infinite in two directions (y, z) and bounded by two parallel metallic plates in a third direction. The distance between these plates is 2a. A modulated current having the form of an infinite layer of thickness 2b, $b < a$, moves in the plasma along the $\vec{z}$ axis. The plasma is located in a constant magnetic field directed along the current, which is so strong that the plasma particle motion across the field may be disregarded.

Let us investigate two problems. In the first problem, let us determine the wave field strength in the linear approximation for an arbitrary dispersion law. In the second problem, let us determine the wave field strength in the nonlinear, but hydrodynamic approximation. /210
We shall assume that the nonlinearity is slight. The following quantity is a small parameter of the problem

$$\varepsilon_\alpha = \frac{e_\alpha E_0}{m_\alpha \omega V_\Phi \left(1 - u_{T\alpha}^2\right)},$$

where e_α and m_α are, respectively, the charge and mass of particles of type α (α = i -- ions, α = e -- electrons); E_0 -- amplitude of wave field strength in the linear approximation; ω -- modulation frequency; v_Φ -- phase velocity of a propagated wave, equalling the velocity of current particles; $u_{T\alpha} = \frac{v_{T\alpha}}{v_\Phi}$ ($v_{T\alpha}$ -- mean thermal velocity of α particles).

We can write the system of equations describing wave excitation in a plasma under the conditions being considered as follows

$$\frac{\partial^2 E}{\partial x^2} + \sigma\omega^2\left(\frac{\partial^2 E}{\partial \xi^2} - E\sum_{\alpha=i}^{e}\frac{\Omega_{0\alpha}^2\omega^1}{\omega^3}\int\frac{V_\Phi^2\frac{\partial f_{0\alpha}}{\partial v}}{V'_\Phi - v}dv\right) + \sigma\omega V_\Phi\sum_{\alpha=i}^{e}\frac{\Omega_{0\alpha}^2 m_\alpha}{e_\alpha\left(1-u_{T\alpha}^2\right)}\times$$
$$\times\left[\left(3-u_{T\alpha}^2\right)\frac{\partial}{\partial\xi}\cdot\frac{u_\alpha^2}{2} + \left(3-2u_{T\alpha}^2\right)\frac{\partial}{\partial\xi}\cdot\frac{u_\alpha^3}{3}\right] = -4\pi\omega\sigma\frac{\partial j}{\partial\xi}; \qquad (1)$$

$$\frac{\partial u_\alpha}{\partial\xi} = \frac{e_\alpha E}{m_\alpha\omega V_\Phi\left(1-u_{T\alpha}^2\right)} + \frac{1+u_{T\alpha}^2}{1-u_{T\alpha}^2}\cdot\frac{\partial}{\partial\xi}\cdot\frac{u_\alpha^2}{2} + \frac{u_{T\alpha}^2}{1-u_{T\alpha}^2}\cdot\frac{\partial}{\partial\xi}\cdot\frac{u_\alpha^3}{3}, \qquad (2)$$

where E - z = component of the wave electric field; $\sigma = \frac{1-\beta^2}{\beta^2 c^2}$ $\left(\beta = \frac{V_\Phi}{c},\right.$ c -- speed of light$\left.\right)$; $\xi = \omega t - k_3 z$; $\Omega_{0\alpha}^2 = \frac{4\pi e_\alpha^2 n_0}{m_\alpha}$ (n_0 -- equilibrium density of plasma particles which is equal for ions and electrons); $\omega' = \omega + i\nu_\alpha$ (ν_α -- frequency of collisions between α particles); $f_{0\alpha}$ -- equilibrium distribution function of α particles normalized to unity; $u_\alpha = \frac{v_\alpha}{V_\Phi}$ (v_α -- hydrodynamic velocity of α particles).

The derivation of equations (1) and (2) is given in (Ref. 4). In these equations, we must set $\gamma = 1$, and we must supplement them with the boundary conditions, which in this case may be reduced to setting the field strength on the wave guide walls equal to zero:

$$E(\pm a) = 0. \qquad (3)$$

We can present the modulated current in the following form /211

$$j = j_1(x)\sin\xi,\quad j_1 = \begin{cases} e\rho_0 V_0, & 0 \leqslant (x) \leqslant b, \\ 0, & b < (x) < a, \end{cases} \qquad (4)$$

where e -- charge; ρ_0 -- density; V_0 -- velocity of current particles.

We can obtain the equation of the linear approximation for the field from equation (1), setting $u_\alpha = 0$ in it. The solution of the linear equation can be written as $E = R(x)\cos\xi$. We obtain the following equation for R(x)

$$\frac{d^2R}{dx^2} + k_\perp^2 R = -4\pi\omega\sigma j_1, \tag{5}$$

where

$$k_\perp^2 = -\sigma\omega^2\left(1 + \sum_{\alpha=i}^{e}\frac{\Omega_{0\alpha}^2\omega'}{\omega^3}\int\frac{V_\Phi^2\frac{\partial f_{0\alpha}}{\partial v}\,dv}{V_\Phi' - v}\right). \tag{5a}$$

Solving equations (5) and (3) together, we find

$$R(x) = \sum_{n=0}^{\infty} R_n \cos\alpha_n x, \tag{6}$$

where

$$R_n = \frac{4\pi\omega\sigma j_n}{\Delta_n};\ j_n = 2\frac{e\rho_0 V_0}{\alpha_n a}\sin\alpha_n b;$$

$$\alpha_n = \frac{\pi}{2a}(2n+1);\ \Delta_n = \alpha_n^2 - k_\perp^2.$$

If the velocity of the current particles is close to the mean thermal velocity of plasma ions or electrons, the integral in formula (5a) must be determined numerically, Im $k_\perp^2$ and Re $k_\perp^2$ are equal in order of magnitude, damping is large, amplitude of the excited wave field strength is small, and the energy losses by the current are insignificant. In this case, we can confine ourselves to the linear theory.

If $V_0 \gg v_{Te,i}$ or $v_{Te} \gg V_0 \gg v_{Ti}$, then Im $k_\perp^2 \ll$ Re $k_\perp^2$ and at a certain frequency of current modulation $|\Delta_n| \sim$ Im $k_\perp^2 \ll \alpha_n$ may hold. The corresponding R_n (we shall designate it by E_0) increases greatly, and only the resonance component can remain in the sum (6): $R(x) \approx E_0\cos\alpha_n x$.

Since the amplitude of the wave field strength is large, the nonlinearity of the medium may be significant. When determining the nonlinear dispersion equation, let us regard the nonlinearity and thermal scatter as small independent additions to the linear hydrodynamic dispersion equation ($\Delta_n = 0$, $\alpha_n^2 = \mathrm{Re}\, k_\perp^2$). Therefore, the wave damping caused by kinetic phenomena can only be taken into consideration in the third approximation. /212

We may find the hydrodynamic velocity of the linear approximation, corresponding to $\Delta_n \approx 0$, from equation (2)

$$u_\alpha^{(1)} = \varepsilon_\alpha \cos \alpha_n x \sin \xi. \tag{7}$$

Let us substitute (7) in equation (1), and let us retain the terms $\sim\varepsilon_\alpha^2$. Then, representing the field E in the second approximation in the form $E^{(2)} = E_0 R_2(x) \sin 2\xi$, we obtain the following equation for $R_2(x)$

$$\frac{d^2 R_2}{dx^2} + q^2 R_2 = f_2 \cos^2 \alpha_n x, \tag{8}$$

where

$$q^2 = -\sigma\omega^2 \left(4 - \sum_{\alpha=i}^{e} \frac{\Omega_{0\alpha}^2}{\omega^2 \left(1 - u_{T\alpha}^2\right)}\right); \; f_2 = -\sigma \sum_{\alpha=i}^{e} \frac{\varepsilon_\alpha}{2} \Omega_{0\alpha}^2 \frac{3 - u_{T\alpha}^2}{\left(1 - u_{T\alpha}^2\right)^2}.$$

Solving equations (8) and (3) together, we find

$$R_2 = \frac{f_2}{2q^2} - \frac{f_2}{q^2} \cdot \frac{2\alpha_n^2}{4\alpha_n^2 - q^2} \cdot \frac{\cos qx}{\cos qa} - \frac{f_2}{2} \cdot \frac{\cos 2\alpha_n x}{4\alpha_n^2 - q^2}. \tag{9}$$

In the second approximation, the hydrodynamic velocity is

$$u_\alpha^{(2)} = -\frac{\varepsilon_\alpha}{2} \left[R_2(x) + \frac{1 + u_{T\alpha}^2}{1 - u_{T\alpha}^2} \cdot \frac{\varepsilon_\alpha}{2} \cos^2 k_\perp x \right] \cos 2\xi. \tag{10}$$

After substituting $u_\alpha^{(1)}$, $u_\alpha^{(2)}$ in the right part of equation (1) of the first and second approximation, we obtain the following equation for the field E in the third approximation

$$\begin{gathered}\frac{\partial^2 E^{(3)}}{\partial x^2} + \sigma\omega^2 \left[\frac{\partial^2 E^{(3)}}{\partial \xi^2} - E^{(3)} \sum_\alpha \frac{\Omega_{0\alpha}^2}{\omega(\omega + i\nu_\alpha)\left(1 - u_{T\alpha}^2\right)}\right] = A_3 \cos 3\psi - \\ - \sigma \sum_\alpha \frac{E_0 \Omega_{0\alpha}^2 \varepsilon_\alpha}{4\left(1 - u_{T\alpha}^2\right)^2} \left\{ \left(3 - u_{T\alpha}^2\right) R_2 + \frac{\varepsilon_\alpha}{2} \frac{9 - 8u_{T\alpha}^2 + 3u_{T\alpha}^4}{1 - u_{T\alpha}^2} \times \right. \\ \left. \times \cos^2 \alpha_n x \right\} \cos \alpha_n x \cos \xi,\end{gathered} \tag{11}$$

where A_3 is a certain function of x which is determined by the third harmonic with respect to ξ.

We can write the solution of equation (11) in the form $E^{(3)} = E_0 R_3(x) \cos\xi$. We obtain the following equation for $R_3(x)$ /213

$$\frac{d^2R_3}{dx^2} + k_\perp^2 R_3 = Q_1 \cos\alpha_n x + Q_2 \cos qx \cos\alpha_n x + Q_3 \cos 3\alpha_n x, \tag{12}$$

where

$$Q_1 = -\sigma \sum_\alpha \frac{\Omega_{0\alpha}^2 \varepsilon_\alpha}{4\left(1-u_{T\alpha}^2\right)^2}\left[\frac{f_2}{4q^2}\left(3-u_{T\alpha}^2\right)\frac{8\alpha_n^2-3q^2}{4\alpha_n^2-q^2} + \frac{3}{8}\varepsilon_\alpha \times \frac{9-8u_{T\alpha}^2+3u_{T\alpha}^4}{1-u_{T\alpha}^2}\right];$$

$$Q_2 = \sigma \sum_\alpha \frac{\Omega_{0\alpha}^2 \varepsilon_\alpha}{4\left(1-u_{T\alpha}^2\right)^2} \cdot \frac{f_2}{q^2}\left(3-u_{T\alpha}^2\right)\frac{2\alpha_n^2}{4\alpha_n^2-q^2} \cdot \frac{1}{\cos qa};$$

$$Q_3 = -\sigma \sum_\alpha \frac{\Omega_{0\alpha}^2 \varepsilon_\alpha}{4\left(1-u_{T\alpha}^2\right)^2}\left[-\frac{f_2}{4} \cdot \frac{3-u_{T\alpha}^2}{4\alpha_n^2-q^2} + \frac{\varepsilon_\alpha}{8} \cdot \frac{9-8u_{T\alpha}^2+3u_{T\alpha}^4}{1-u_{T\alpha}^2}\right].$$

Since $Q_{1,2,3} \sim \varepsilon^2$, we can write the solution of equation (12) in the following form (Ref. 5)

$$R_3 = \cos\psi + B_1 \cos(\alpha_n + q)x + B_2 \cos(\alpha_n - q)x + B_3 \cos 3\alpha_n x,$$
$$\frac{d\psi}{dx} = k_\perp + \varepsilon^2 A.$$

We thus find

$$R_3 = \cos\left(k_\perp - \frac{Q_1}{2k_\perp}\right)x - \frac{Q_2}{2q}\left[\frac{\cos(\alpha_n+q)x}{2\alpha_n+q} - \frac{\cos(\alpha_n-q)x}{2\alpha_n-q}\right] - \frac{Q_3}{8\alpha_n^2}\cos 3\alpha_n x.$$

Employing the boundary condition (3), we find that

$$\cos\left(k_\perp - \frac{Q_1}{2k_\perp}\right)a - (-1)^n \frac{Q_2}{q} \cdot \frac{2\alpha_n \sin qa}{4\alpha_n^2 - q^2} = 0. \tag{13}$$

We can write the solution of equation (13) as follows

$$k_\perp = \alpha_n + \delta, \quad \delta \sim \varepsilon^2 \ll \alpha_n.$$

We find

$$2\alpha_n\delta = Q_1 + \frac{4\alpha_n^2 Q_2}{4\alpha_n^2 - q^2} \cdot \frac{\sin qa}{qa}. \tag{14}$$

Thus, we can write the nonlinear hydrodynamic dispersion equation as follows /214

$$\sigma\omega^2\left[-1+\sum_\alpha \frac{\Omega_{0\alpha}^2}{\omega(\omega+i\nu_\alpha)\left(1-u_{T\alpha}^2\right)}\right]=\alpha_n^2+2\alpha_n\delta, \tag{15}$$

and ν_α includes both pair collisions and Cherenkov wave absorption by plasma particles.

In the case of resonance, when $\mathrm{Re}\,k_\perp^2 = \alpha_n^2$, the field strength amplitude of the excited wave is

$$E_0=\frac{-4\pi\omega\sigma j_n}{\left[(2\alpha_n\delta)^2+\left(\sigma\sum_\alpha \frac{\Omega_{0\alpha}^2}{1-u_{T\alpha}^2}\cdot\frac{\nu_\alpha}{\omega}\right)^2\right]^{1/2}}. \tag{16}$$

Let us first examine high frequency oscillations: $u_{Ti,e}^2 \ll 1$. Disregarding the electron mass as compared with the ion mass, we obtain

$$2\alpha_n\delta=-\sigma\varepsilon_e^2\Omega_{0e}^2\delta_1;\ \delta_1=\frac{3}{8}\cdot\frac{10\omega^2-\Omega_{0e}^2}{4\omega^2-\Omega_{0e}^2}-\frac{\left(\Omega_{0e}^2-\omega^2\right)^2}{\Omega_{0e}^2\left(4\omega^2-\Omega_{0e}^2\right)}\cdot\frac{\mathrm{tg}\,qa}{qa}. \tag{17}$$

We can show that in the case of $\omega \lessgtr \Omega_{0e}$, $\delta_1 > 0$ always holds. We obtain the following value from the equation (15) for the square of the phase velocity

$$\beta^2=\frac{1}{\Omega_{0e}^2-\omega^2+c^2\alpha_n^2}\left[\Omega_{0e}^2-\omega^2+\frac{\varepsilon_e^2\delta_1\alpha_n^2c^2\Omega_{0e}^2}{\Omega_{0e}^2-\omega^2+c^2\alpha_n^2}\right]. \tag{18}$$

Since $\delta_1 > 0$, the phase velocity increases with an increase in the field strength amplitude of the wave. We may use formula (16) to determine the amplitude of the field strength:

$$E_0=-\frac{8\pi\omega e p_0 V_0}{\Omega_{0e}^2\left[\left(\varepsilon_e^2\delta_1\right)^2+\left(\frac{\nu_e}{\omega}\right)^2\right]^{1/2}}\cdot\frac{\sin\alpha_n b}{\alpha_n a}. \tag{19}$$

If $\varepsilon^2\delta_1 > \frac{\nu_e}{\omega}$, the maximum amplitude is determined by the nonlinearity:

$$E_0^3=-\frac{8\pi m_e^2\omega^3V_0^3p_0}{e\Omega_{0e}^2\delta_1}\cdot\frac{\sin\alpha_n b}{\alpha_n a}. \tag{20}$$

The low frequency oscillations: $u_{Te} >> 1 >> u_{Ti}$.

The amplitude of the wave field strength in this case is /215

$$E_0 = \frac{8\pi\omega e\rho_0 V_0}{\Omega_{0i}^2\left[\left(\varepsilon_i^2\delta_2\right)^2 + \left(\frac{\nu_i}{\omega}\right)^2\right]^{1/2}} \cdot \frac{\sin \alpha_n b}{\alpha_n a}. \tag{21}$$

If $u_{Te}^2\mu \gg 1 \left(\mu = \frac{m_e}{m_i}\right)$, then δ_2 and the phase velocity have the values of (17) and (18), respectively, for a replacement of the indices $e \to i$. In the case of sound oscillations, when $\omega^2 \ll \frac{\Omega_{0i}^2}{\mu u_{Te}^2}$, we have

$$\delta_2 = \frac{3}{32}\left(9 - 5\frac{\sigma\Omega_{0i}^2}{q^2}\right) + \frac{\sigma\Omega_{0i}^2}{q^2} \cdot \frac{\operatorname{tg} qa}{qa}; \tag{22}$$

$$V_\Phi^2 = \mu v_{Te}^2 \frac{\Omega_{0i}^2\left(1 + \varepsilon_i^2\delta_2\right)}{\Omega_{0i}^2 + \alpha_n^2\mu v_{Te}^2}. \tag{23}$$

If $\frac{\nu_i}{\omega} < \varepsilon_i^2\delta_2$, the field strength amplitude is limited by the nonlinearity and has the same form as in the case of high frequency oscillations (2), with a replacement of the indices $e \to i$, $1 \to 2$.

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